butterflies

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Contents

1 butterflies.cabal

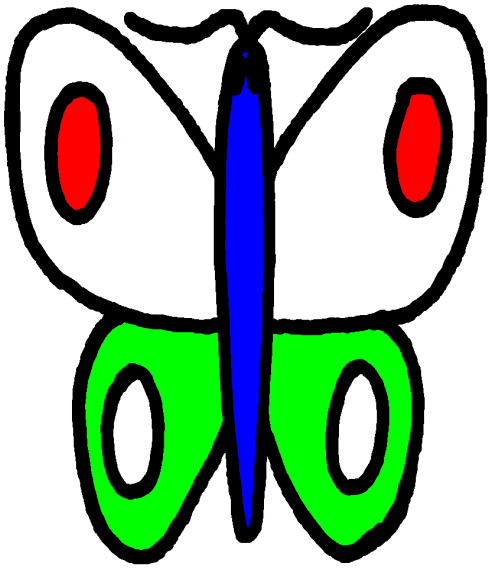
```
name:                butterflies
version:             0.3.0.2
synopsis:            butterfly tilings
description:         various tilings with butterflies (after M C Escher 1950)
homepage:            https://code.mathr.co.uk/butterflies
license:             GPL-3
license-file:        LICENSE
author:              Claude Heiland-Allen
maintainer:          claude@mathr.co.uk
copyright:           (C) 2013,2015,2018,2019  Claude Heiland-Allen <claude@mathr
    ↪ .co.uk>
category:            Graphics
build-type:          Simple
cabal-version:       >=1.8
```

```
15  extra-source-files:  README
    data-dir:            data
    data-files:
      butterfly.png
      palette.png
20
    library
      exposed-modules:
        Geometry.Class
        Geometry.Flat.ThreeD.Space
25      Geometry.Flat.TwoD.Space
        Geometry.Flat.TwoD.Tessellation.Triangular
        Geometry.Hyperbolic.TwoD.HalfSpace
        Geometry.Utils
      build-depends:
30      base >=4.6 && <4.14

    executable butterflies-flat
      hs-source-dirs:      flat
      main-is:             Main.hs
35    other-modules:
        Paths_butterflies
      build-depends:
        base,
        bytestring >=0.10 && <0.11,
40      OpenGLRaw >=3.3 && <3.4,
        GLUT >=2.7 && <2.8,
        gl-capture >=0.1 && <0.2,
        repa >=3.4 && <3.5,
        repa-devil >=0.3 && <0.4,
45      butterflies

    source-repository head
      type:                git
      location:            https://code.mathr.co.uk/butterflies.git
50
    source-repository this
      type:                git
      location:            https://code.mathr.co.uk/butterflies.git
      tag:                 v0.3.0.2
```

2 data/butterfly.png



3 data/palette.png

4 flat/diagram.hs

```
import Diagrams.Prelude
import Diagrams.Backend.Cairo.CmdLine (Cairo, defaultMain)

import Control.Monad (forM_)
import System.Environment (withArgs)

main :: IO ()
main = do
  forM_ [2 .. 5] $ \s -> forM_ [0 .. s `div` 2] $ \q -> do
    let p = s - q
        n = p * p + q * q + p * q
    withArgs ["-o", "colouring-" ++ (if n < 10 then "0" else "") ++ show n ++ "-" ++ show p ++ "-" ++ show q ++ ".png"] $
      defaultMain (diagram p q)

diagram :: Integer -> Integer -> QDiagram Cairo R2 Any
diagram p q = dia
  where
    n = p * p + q * q + p * q
    i = p `gcd` q
    j = n `div` i
    o = head [ o' | o' <- [0 .. j - 1], (p + q) `mod` j == (o' * i * (q `div` i)) `mod` j ]
    s = 32
    t = s * sqrt 3 / 2
```

```
      x = -100
      y = -50
      w = 200
      h = 100
      pt u v = p2 (s * (fromInteger u + fromInteger v / 2), t * fromInteger v)
      rt u v = r2 (s * (fromInteger u + fromInteger v / 2), t * fromInteger v)
      triangle u v a b = close $ fromVertices [ pt u v, pt (u + b) (v + a), pt (u - a) (v + b + a) ]
      grid = mconcat [ triangle (u - z) v 1 0 | v <- [y .. y + h], let z = (v + 1) `div` 2, u <- [x .. x + w] ]
      pargram u v = translate (r2(-3*s/4, -t/2)) $ close $ fromVertices [ pt u v, pt (u + j) v, pt (u + j) (v + i), pt u (v + i) ]
      isV u v = local u v == (0, 0)
      local u v =
        let d = i * (v `div` i)
        in  ((u + o * d) `mod` j, v `mod` i)
      border uv d =
        let xx = strutX (uv * s)
            yy = strutY (uv * t)
        in  beside (r2 (0,-1)) (beside (r2(0,1)) (beside (r2(-1,0)) (beside (r2 (1,0)) d xx) xx) yy) yy
      (uu, vv) = local 1 (-1)
      pp = 1 - uu
      qq = -1 - vv
      dia0 = mconcat
        [ stroke (fromVertices [pt 0 0, pt (-p) 0, pt (-(p + q)) q]) # lc black # lw 3 # border 5
        , stroke (triangle 0 0 p q)  # fc red  # opacity 0.5 # border 5
        , stroke (pargram pp qq) # fc blue # opacity 0.5 # border 5
        ]
      dia = mconcat
        [ mconcat [ stroke (translate (rt u v) $ circle (s / 4)) # lc black # lw 1 | v <- [y .. y + h], u <- [x .. x + w], isV u v ]
        , mconcat [ translate (rt u v) $ circle (s / 4) | u <- [pp .. pp + j - 1], v <- [qq .. qq + i - 1] ] # fc yellow
        , dia0
        , mconcat [ stroke (triangle u v p q) # lc red  # lw 1 | v <- [y .. y + h], u <- [x .. x + w], isV u v ]
        , mconcat [ stroke (pargram u v)      # lc blue # lw 1 | v <- [y .. y + h], u <- [x .. x + w], isV u v ]
        , stroke grid # lc green # lw 1
        , rect (s * fromIntegral w) (t * fromIntegral h) # fc white
        ] # withEnvelope dia0 # centerXY # withEnvelope (rect 1024 576 `asTypeOf` dia)
```

5 flat/Main.hs

```
{-# LANGUAGE TypeSynonymInstances #-}
module Main (main) where

import Control.Monad (when)
import Foreign (Ptr, nullPtr, castPtr, plusPtr, advancePtr, allocaBytes, with, withArray, peek, poke, pokeByteOff)
import Foreign.C (CUChar)
import Foreign.C.String (withCString, peekCString)
import Foreign.ForeignPtr (withForeignPtr)
import System.Exit (exitSuccess)
```

```
10
    import Graphics.GL
    import Graphics.Rendering.OpenGL.Capture (capturePPM)
    import Graphics.UI.GLUT (getArgsAndInitialize, createWindow, displayCallback, ($~
        ↳ =), swapBuffers, reportErrors, mainLoop)
    import qualified Data.ByteString as BS
15  import Data.Array.Repa.IO.DevIL (runIL, readImage, Image(RGBA))
    import Data.Array.Repa.Repr.ForeignPtr (toForeignPtr)

    import Paths_butterflies (getDataFileName)

20  import Geometry.Flat.TwoD.Space
    import Geometry.Flat.TwoD.Tessellation.Triangular

    swarm' p q = tessellate p q 3 (Point (-12) (-12)) (Point 12 12)

25  class GLPoke t where glPoke :: t -> Ptr t -> IO (Ptr t)
    instance GLPoke t => GLPoke [t] where
      glPoke [] p = return p
      glPoke (x:xs) p = glPoke x (castPtr p) >>= glPoke xs . castPtr

30  instance GLPoke GLfloat where glPoke x p = poke p x >> return (advancePtr p 1)
    instance GLPoke Double where
      glPoke x p = do
        p <- glPoke (realToFrac x :: GLfloat) (castPtr p)
        return (castPtr p)
35  instance GLPoke Point where
      glPoke (Point x y) p = do
        p <- glPoke x (castPtr p)
        p <- glPoke y p
        return (castPtr p)
40  instance GLPoke Vertex where
      glPoke (Vertex vp vt h0 h1 h2) p = do
        p <- glPoke [vp, vt] (castPtr p)
        p <- glPoke [h0, h1, h2] (castPtr p)
        return (castPtr p)
45  instance GLPoke a => GLPoke (Triangle a) where
      glPoke (Triangle a b c) p = do
        p <- glPoke [a,b,c] (castPtr p)
        return (castPtr p)

50  vert = unlines
      [ "#version 120"
      , "uniform mat4 mvp;"
      , "attribute vec2 p0;"
      , "attribute vec2 t0;"
55    , "attribute vec3 c0;"
      , "varying vec2 t;"
      , "varying vec3 c;"
      , "void main() {"
      , "  gl_Position = vec4(vec2(vec4(p0, 0.0, 1.0) * mvp), 0.0, 1.0);"
60    , "  t = t0;"
      , "  c = c0;"
      , "}"
      ]

65  frag = unlines
```

```
  [ "#version 120"
  , "uniform sampler2D tex;"
  , "uniform sampler1D pal;"
  , "varying vec2 t;"
  , "varying vec3 c;"
  , "const float phi1 = (sqrt(5.0) - 1.0) / 2.0;"
  , "const float phi2 = (sqrt(5.0) - 2.0) / 2.0;"
  , "vec4 colour(float i) {"
  , "  float j = 1.0/6.0 - phi1 * i;"
  , "  float k = phi2 * i;"
  , "  j -= floor(j);"
  , "  k -= floor(k);"
  , "  k *= -0.5;"
  , "  k +=  1.0;"
  , "  return vec4(texture1D(pal, j).rgb * k, 1.0);"
  , "}"
  , "void main() {"
  , "  vec4 w = texture2D(tex, t);"
  , "  vec4 f = vec4(0.0);"
  , "  if (w.a <= 0.5) {"
  , "    f = vec4(0.5, 0.5, 0.5, 1.0);"
  , "  } else if (w.r >= 0.5 && w.g >= 0.5 && w.b >= 0.5) {"
  , "    f = vec4(1.0, 1.0, 1.0, 1.0);"
  , "  } else if (w.r <= 0.5 && w.g <= 0.5 && w.b <= 0.5) {"
  , "    f = vec4(0.0, 0.0, 0.0, 1.0);"
  , "  } else {"
  , "    float k = 0.0;"
  , "    if (w.r >= 0.5) {"
  , "      k = c.r;"
  , "    } else if (w.g >= 0.5) {"
  , "      k = c.g;"
  , "    } else if (w.b >= 0.5) {"
  , "      k = c.b;"
  , "    }"
  , "    f = colour(k);"
  , "  }"
  , "  gl_FragColor = f;"
  , "}"
  ]

main = do
  (_, [sp, sq]) <- getArgsAndInitialize
  _ <- createWindow "butterflies"
  program <- compileProgram
  glUseProgram program
  loadTextures
  let ip = read sp
      iq = read sq
      swarm = swarm' ip iq
      count = length swarm * 3
      stride = 4 * (2 + 2 + 3)
      bytes = count * fromIntegral stride
  allocaBytes bytes $ \p -> do
    glPoke swarm p
    vbo <- with 0 $ \q -> glGenBuffers 1 q >> peek q
    glBindBuffer GL_ARRAY_BUFFER vbo
    glBufferData GL_ARRAY_BUFFER (fromIntegral bytes) p GL_STATIC_DRAW
```

```
    att <- withCString "p0" $ glGetAttribLocation program
    glVertexAttribPointer (fromIntegral att) 2 GL_FLOAT (fromIntegral GL_FALSE) ↙
        ↳ stride (plusPtr nullPtr 0)
    glEnableVertexAttribArray (fromIntegral att)
    att <- withCString "t0" $ glGetAttribLocation program
    glVertexAttribPointer (fromIntegral att) 2 GL_FLOAT (fromIntegral GL_FALSE) ↙
        ↳ stride (plusPtr nullPtr (2 * 4))
    glEnableVertexAttribArray (fromIntegral att)
    att <- withCString "c0" $ glGetAttribLocation program
    glVertexAttribPointer (fromIntegral att) 3 GL_FLOAT (fromIntegral GL_FALSE) ↙
        ↳ stride (plusPtr nullPtr (4 * 4))
    glEnableVertexAttribArray (fromIntegral att)
  let s = 10
      l = -s
      r = s
      t = -s
      b = s
      n = -1
      f = 1
      ortho = [ 2 / (r - l), 0, 0, -(r + l) / (r - l)
              , 0, 2 / (t - b), 0, -(t + b) / (t - b)
              , 0, 0, 2 / (f - n), -(f + n) / (f - n)
              , 0, 0, 0, 1 ]
  withArray ortho $ \p -> do
    loc <- withCString "mvp" $ glGetUniformLocation program
    glUniformMatrix4fv loc 1 (fromIntegral GL_FALSE) p
  loc <- withCString "tex" $ glGetUniformLocation program
  glUniform1i loc 0
  loc <- withCString "pal" $ glGetUniformLocation program
  glUniform1i loc 1
  glClearColor 0.5 0.5 0.5 1
  displayCallback $= do
    glClear GL_COLOR_BUFFER_BIT
    glDrawArrays GL_TRIANGLES 0 (fromIntegral count)
    swapBuffers
    let ipq = ip * ip + iq * iq + ip * iq
    capturePPM >>= BS.writeFile ("tessellation-" ++ (if ipq < 10 then "0" else ↙
        ↳ "") ++ show ipq ++ "-" ++ show ip ++ "-" ++ show iq ++ ".ppm")
    reportErrors
    exitSuccess
  mainLoop

loadTextures = do
  RGBA img <- getDataFileName "butterfly.png" >>= runIL . readImage
  withForeignPtr (toForeignPtr img) $ \p -> do
    tex <- with 0 $ \q -> glGenTextures 1 q >> peek q
    glBindTexture GL_TEXTURE_2D tex
    glTexImage2D GL_TEXTURE_2D 0 (fromIntegral GL_RGBA) 1024 1024 0 GL_RGBA ↙
        ↳ GL_UNSIGNED_BYTE p
    glTexParameteri GL_TEXTURE_2D GL_TEXTURE_MIN_FILTER (fromIntegral ↙
        ↳ GL_LINEAR_MIPMAP_LINEAR)
    glTexParameteri GL_TEXTURE_2D GL_TEXTURE_MAG_FILTER (fromIntegral GL_LINEAR)
    glGenerateMipmap GL_TEXTURE_2D
  glActiveTexture GL_TEXTURE1
  RGBA img <- getDataFileName "palette.png" >>= runIL . readImage
  withForeignPtr (toForeignPtr img) $ \p -> do
    tex <- with 0 $ \q -> glGenTextures 1 q >> peek q
```

```
    glBindTexture GL_TEXTURE_1D tex
    glTexImage1D GL_TEXTURE_1D 0 (fromIntegral GL_RGBA) 256 0 GL_RGBA ↙
        ↳ GL_UNSIGNED_BYTE p
    glTexParameteri GL_TEXTURE_1D GL_TEXTURE_MIN_FILTER (fromIntegral GL_NEAREST↙
        ↳ )
    glTexParameteri GL_TEXTURE_1D GL_TEXTURE_MAG_FILTER (fromIntegral GL_NEAREST↙
        ↳ )

compileProgram = do
  program <- glCreateProgram
  compileShader program GL_VERTEX_SHADER vert
  compileShader program GL_FRAGMENT_SHADER frag
  glLinkProgram program
  debugProgram program
  return program

compileShader program t src = do
  shader <- glCreateShader t
  withCString src $ \srcp -> with srcp $ \srcpp -> glShaderSource shader 1 srcpp↙
      ↳ nullPtr
  glCompileShader shader
  glAttachShader program shader
  debugShader shader
  glDeleteShader shader

debugProgram program = do
  if program /= 0
    then do
      linked <- with 0 $ \p -> glGetProgramiv program GL_LINK_STATUS p >> peek p
      when (linked /= fromIntegral GL_TRUE) $ putStrLn "link failed"
      len <- with 0 $ \p -> glGetProgramiv program GL_INFO_LOG_LENGTH p >> peek ↙
          ↳ p
      when (len > 1) $ do
        allocaBytes (fromIntegral len + 1) $ \p -> glGetProgramInfoLog program ↙
            ↳ len nullPtr p >> pokeByteOff p (fromIntegral len) (0 :: CUChar) >>↙
            ↳ peekCString p >>= putStrLn
    else putStrLn "no program"

debugShader shader = do
  if shader /= 0
    then do
      compiled <- with 0 $ \p -> glGetShaderiv shader GL_COMPILE_STATUS p >> ↙
          ↳ peek p
      when (compiled /= fromIntegral GL_TRUE) $ putStrLn "compile failed"
      len <- with 0 $ \p -> glGetShaderiv shader GL_INFO_LOG_LENGTH p >> peek p
      when (len > 1) $ do
        allocaBytes (fromIntegral len + 1) $ \p -> glGetShaderInfoLog shader len↙
            ↳ nullPtr p >> pokeByteOff p (fromIntegral len) (0 :: CUChar) >> ↙
            ↳ peekCString p >>= putStrLn
    else putStrLn "no shader"
```

6 Geometry/Class.hs

```
{-# LANGUAGE MultiParamTypeClasses, FunctionalDependencies #-}
module Geometry.Class where

class Geometry point geodesic | point -> geodesic, geodesic -> point where
```

```
    distance :: point -> point -> Double
    angle :: point -> point -> point -> Double
    geodesic :: point -> point -> geodesic
    rotate :: Double -> point -> point -> point
    translate :: Double -> geodesic -> point -> point
    midpoint :: point -> point -> point
    midpoint p q = translate (distance p q / 2) (geodesic p q) p

class Embedding model point where
  embed :: point -> model
  model :: model -> Maybe point
```

7 Geometry/Flat/ThreeD/Space.hs

```
{-# LANGUAGE MultiParamTypeClasses #-}
module Geometry.Flat.ThreeD.Space
  ( Point(..)
  , Geodesic(..)
  ) where

import Geometry.Class
import qualified Geometry.Flat.TwoD.Space as E

data Point = Point{ pX, pY, pZ :: !Double } deriving Show
data Geodesic = Geodesic{ gO :: !Point, gX, gY, gZ :: !Double } deriving Show

instance Geometry Point Geodesic where

  distance (Point u v w) (Point x y z) =
    let dx = x - u
        dy = y - v
        dz = z - w
    in  sqrt (dx * dx + dy * dy + dz * dz)

  angle _ _ _ = error "Geometry.Flat.ThreeD.Space.angle"

  geodesic p@(Point u v w) (Point x y z) =
    let dx = x - u
        dy = y - v
        dz = z - w
        d = sqrt (dx * dx + dy * dy + dz * dz)
    in  Geodesic p (dx / d) (dy / d) (dz / d)

  rotate _ _ _ = error "Geometry.Flat.ThreeD.Space.rotate"

  translate d (Geodesic _ dx dy dz) (Point x y z) =
    Point (x + d * dx) (y + d * dy) (z + d * dz)

  midpoint (Point u v w) (Point x y z) =
    Point (0.5 * (u + x)) (0.5 * (v + y)) (0.5 * (w + z))

instance Embedding Point Point where
  embed = id
  model = Just

instance Embedding Point E.Point where
  embed (E.Point x y) = Point x y 0
```

```
  model (Point x y z)
    | z == 0 = Just (E.Point x y)
    | otherwise = Nothing
```

8 Geometry/Flat/TwoD/Space.hs

```
{-# LANGUAGE MultiParamTypeClasses #-}
module Geometry.Flat.TwoD.Space
  ( Point(..)
  , Geodesic(..)
  ) where

import Geometry.Class
import Geometry.Utils (fixAngle)

data Point = Point{ pX, pY :: !Double } deriving Show
data Geodesic = Geodesic{ gO :: !Point, gX, gY :: !Double } deriving Show

instance Geometry Point Geodesic where

  distance (Point u v) (Point x y) =
    let dx = x - u
        dy = y - v
    in  sqrt (dx * dx + dy * dy)

  angle (Point p q) (Point u v) (Point x y) =
    let du = u - p
        dv = v - q
        dx = x - p
        dy = y - q
        uv = atan2 dv du
        xy = atan2 dy dx
    in  fixAngle (xy - uv)

  geodesic p@(Point u v) (Point x y) =
    let dx = x - u
        dy = y - v
        d = sqrt (dx * dx + dy * dy)
    in  Geodesic p (dx / d) (dy / d)

  rotate a (Point u v) (Point x y) =
    let co = cos a
        si = sin a
        p = x - u
        q = y - v
    in  Point (u + co * p - si * q) (v + si * p + co * q)

  translate d (Geodesic _ dx dy) (Point x y) = Point (x + d * dx) (y + d * dy)

  midpoint (Point u v) (Point x y) = Point (0.5 * (u + x)) (0.5 * (v + y))

instance Embedding Point Point where
  embed = id
  model = Just
```

9 Geometry/Flat/TwoD/Tessellation/Triangular.hs

```
module Geometry.Flat.TwoD.Tessellation.Triangular
  ( tessellate
  , Triangle(..)
  , Vertex(..)
  ) where

import Geometry.Flat.TwoD.Space

data Triangle a = Triangle a a a deriving Show

data Vertex = Vertex{ vPos, vTex :: !Point, vSpots, vWings, vBody :: !Double }
  deriving Show

tessellate :: Int -> Int -> Double -> Point -> Point -> [Triangle Vertex]
tessellate p q s (Point lx ly) (Point hx hy) =
  let t = s * sqrt 3 / 2
      y = floor (ly / t)
      x = floor (lx / s)
      w = ceiling ((hx - lx) / s)
      h = ceiling ((hy - ly) / t)
      g = grid p q
      pc u v = (Point (s * (fromIntegral u + fromIntegral v / 2)) (t * ↙
        ↳ fromIntegral v), g u v)
      b (p0@(Point x0 y0), h0) (Point x1 y1, h1) (Point x2 y2, h2) =
        let pTL = p0
            pTM = Point ((2 * x0 + x1) / 3) ((2 * y0 + y1) / 3)
            pTR = Point ((x0 + 2 * x1) / 3) ((y0 + 2 * y1) / 3)
            pBL = Point ((2 * x0 + x2) / 3) ((2 * y0 + y2) / 3)
            pBR = Point ((x0 + x1 + x2) / 3) ((y0 + y1 + y2) / 3)
            tTL = Point 0    1
            tTM = Point 0.5  1
            tTR = Point 1    1
            tBL = Point 0.25 0
            tBR = Point 0.75 0
            v pp tt = Vertex pp tt h0 h2 h1
            vTL = v pTL tTL
            vTM = v pTM tTM
            vTR = v pTR tTR
            vBL = v pBL tBL
            vBR = v pBR tBR
        in  [ Triangle vTL vTM vBL
            , Triangle vTM vTR vBR
            , Triangle vBL vTM vBR
            ]
      bs v0 v1 v2 = concat
        [ b v0 v1 v2
        , b v1 v2 v0
        , b v2 v0 v1
        ]
      tris u v = concat
        [ bs (pc u v) (pc (u+1) v) (pc u (v+1))
        , bs (pc (u+1) v) (pc (u+1) (v+1)) (pc u (v+1))
        ]
  in  concat
        [ tris (u - z) v
        | v <- [y .. y + h]
        , let z = (v + 1) `div` 2
```

```
        , u <- [x .. x + w]
        ]

grid :: Int -> Int -> Int -> Int -> Double
grid p q =
  let n = p * p + q * q + p * q
      i = p `gcd` q
      j = n `div` i
      o = head [ o' | o' <- [0 .. j - 1], (p + q) `mod` j == (o' * i * (q `div` ↙
        ↳ i)) `mod` j ]
  in \x y ->
      let d = i * (y `div` i)
      in  fromIntegral (((x + o * d) `mod` j) + j * (y `mod` i))
```

10 Geometry/Hyperbolic/TwoD/HalfSpace.hs

```
{-# LANGUAGE MultiParamTypeClasses #-}
module Geometry.Hyperbolic.TwoD.HalfSpace
  ( Point(..)
  , Geodesic(..)
  ) where

import Data.Fixed (mod')

import Geometry.Class
import qualified Geometry.Flat.TwoD.Space as E

data Point = Point{ pX, pY :: !Double } deriving Show

data Geodesic
  = Line{ gDir :: !Bool, gCX :: !Double }
  | Arc { gDir :: !Bool, gCX :: !Double, gR :: !Double }
  deriving Show

instance Geometry Point Geodesic where

  distance p q = case geodesic p q of
    Line{} -> abs (log (pY p / pY q))
    g@Arc{} ->
      let a = gCX g - gR g
          b = gCX g + gR g
          pa = sqrt ((pX p - a) * (pX p - a) + pY p * pY p)
          pb = sqrt ((pX p - b) * (pX p - b) + pY p * pY p)
          qa = sqrt ((pX q - a) * (pX q - a) + pY q * pY q)
          qb = sqrt ((pX q - b) * (pX q - b) + pY q * pY q)
      in  abs (log ((pa / pb) / (qa / qb)))

  angle _ _ _ = error "Geometry.Hyperbolic.TwoD.HalfPlane.angle"

  geodesic p q
    | abs (pX p - pX q) < eps =
          Line{ gCX = 0.5 * (pX p + pX q), gDir = pY p < pY q }
    | otherwise =
        let cx = 0.5 * (pX q * pX q + pY q * pY q - pX p * pX p - pY p * pY p) /↙
            ↳  (pX q - pX p)
            r = sqrt ((pX p - cx) * (pX p - cx) + pY p * pY p)
        in  Arc{ gCX = cx, gDir = pX p < pX q, gR = r }
```

```
  rotate a p q =
    let g = geodesic p q
        b = angleAt g p
        h = fromPointAngle p (b + a)
        d = distance p q
    in  translate d h p

  translate d g p = atDist g p (abs d) (d >= 0)

instance Embedding Point Point where
  embed = id
  model = Just

instance Embedding E.Point Point where
  embed (Point x y) = E.Point x y
  model (E.Point x y)
    | y > 0     = Just (Point x y)
    | otherwise = Nothing

eps :: Double
eps = 1e-12

atParam :: Geodesic -> Double -> Point
atParam g@Line{} t = Point (gCX g) t
atParam g@Arc {} t = Point (gR g * cos (pi - t) + gCX g) (gR g * sin t)

paramAt :: Geodesic -> Point -> Double
paramAt   Line{} p = pY p
paramAt g@Arc {} p = pi - atan2 (pY p) (pX p - gCX g)

angleAt :: Geodesic -> Point -> Double
angleAt g@Line{} _ = (if gDir g then (subtract pi) else id) (pi/2)
angleAt g@Arc {} p = (if gDir g then (pi +) else id) (atan2 (pY p) (pX p - gCX g↙
  ↳ ) + pi/2)

fromPointAngle :: Point -> Double -> Geodesic
fromPointAngle p a
  | abs ((a `mod` pi) - pi/2) < eps =
        Line{ gCX = pX p, gDir = cos a >= 0 }
  | otherwise =
      let b = a + pi/2
          co = cos b
          si = sin b
          r = abs (pY p / si)
          cx = -(pY p * co - pX p * si) / si
      in  Arc{ gCX = cx, gDir = cos a >= 0, gR = r }

atDist :: Geodesic -> Point -> Double -> Bool -> Point
atDist g@Line{} p d dir
  | dir /= gDir g = binarySearch g p d (0 + eps) pp  (<)
  | otherwise     = binarySearch g p d pp (pp * 100) (>)
  where pp = paramAt g p
atDist g@Arc {} p d dir
  | dir /= gDir g = binarySearch g p d (0 + eps) pp  (<)
  | otherwise     = binarySearch g p d pp (pi - eps) (>)
  where pp = paramAt g p
```

```
binarySearch :: Geodesic -> Point -> Double -> Double -> Double -> (Double -> ↙
    ↳ Double -> Bool) -> Point
binarySearch g p d lo0 hi0 cmp = go lo0 hi0
  where
    go lo hi =
      let mid = 0.5 * (lo + hi)
          q = atParam g mid
          m = distance p q
      in  if hi - lo < eps
            then q
            else if m `cmp` d
                   then go lo mid
                   else go mid hi
```

11 Geometry/Utils.hs

```
module Geometry.Utils
  ( fixAngle
  ) where

import Data.Fixed (mod')

fixAngle :: Double -> Double
fixAngle a =
  let b = a `mod'` (2 * pi)
  in  if b > pi then b - 2 * pi else b
```

12 .gitignore

```
.cabal-sandbox
cabal.sandbox.config
dist
*.ppm
```

13 hyperbolic/Colour.hs

```
module Colour
  ( colour
  ) where

import Control.Monad (liftM2)
import Data.List (partition)
import Data.Maybe (fromMaybe, listToMaybe)

import HalfPlane
import Trapezium
import Fuzz as F

p', q', hyp, adj, opp, d0, d1 :: Double
p' = 7
q' = 3
hyp = acosh ( (cos (pi/p') * cos (pi/q')) / (sin (pi/p') * sin (pi/q')) )
adj = acosh ( cos (pi/q') / sin (pi/p') )
opp = acosh ( cos (pi/p') / sin (pi/q') )
d0 = 2 * hyp + 2 * adj + 2 * opp
```

```
d1 = 2 * acosh ( cos (pi / 7) / sin (pi / 14) )

vertices :: [Trapezium] -> [(Int, Fuzz Point)]
vertices
  = zip [0..23]
  . map (F.fromList (equivalent 0))
  . concatMap (equivalenceClassesBy (equivalent d1))
  . equivalenceClassesBy (equivalent d0)
  . map snd
  . F.toList
  . F.fromList (equivalent 0)
  . liftM2 ($) [bTL, bFR, bFL]

colour' :: [(Int, Fuzz Point)] -> Trapezium -> Trapezium
colour' fs t = t{ bSpots = cTL, bWings = cFL, bBody = cFR }
  where
    cTL = go bTL
    cFL = go bFL
    cFR = go bFR
    go k = fromMaybe 12 $ listToMaybe [ c | (c, f) <- fs, k t `F.elem` f ]

colour :: [Trapezium] -> [Trapezium]
colour ts = let fs = vertices ts in map (colour' fs) ts

equivalent :: Double -> Point -> Point -> Bool
equivalent d p q = abs ((p `dist` q) - d) < 1e-3

equivalenceClasses :: Eq a => [a] -> [[a]]
equivalenceClasses = equivalenceClassesBy (==)

equivalenceClassesBy :: (a -> a -> Bool) -> [a] -> [[a]]
equivalenceClassesBy eq zs = go [] [] zs
  where
    go cs []     [] = cs
    go [] [] (x:xs) = go [[x]] [] xs
    go cs (o:os) [] = go ([o]:cs) [] os
    go (c:cs) os is = let f i = any (`eq` i) c
                          (is',os') = partition f is
                      in if null is'
                          then go (c:cs) (os' ++ os) []
                          else go ((is' ++ c) : cs) [] (os' ++ os)
    go [] (_:_) (_:_) = error "equivalenceClassesBy invariant violated"
```

14 hyperbolic/Fuzz.hs

```
module Fuzz
  ( Fuzz()
  , empty
  , size
  , insert
  , lookup
  , elem
  , delete
  , toList
  , fromList
  ) where
```

```
import Prelude hiding (elem, lookup)
import Data.List (foldl', partition)
import Data.Maybe (isJust, listToMaybe)

data Fuzz a = Fuzz
  { _size   :: Integer
  , _insert :: a -> (Integer, Fuzz a)
  , _lookup :: Integer -> Maybe a
  , _elem   :: a -> Bool
  , _delete :: a -> Fuzz a
  , _toList :: [(Integer, a)]
  }

empty :: (a -> a -> Bool) -> Fuzz a
empty eq = fuzz eq 0 0 []

fromList :: (a -> a -> Bool) -> [a] -> Fuzz a
fromList eq = foldl' (\f a -> snd (insert f a)) (empty eq)

size :: Fuzz a -> Integer
size = _size

insert :: Fuzz a -> a -> (Integer, Fuzz a)
insert = _insert

lookup :: Fuzz a -> Integer -> Maybe a
lookup = _lookup

elem :: a -> Fuzz a -> Bool
elem = flip _elem

delete :: Fuzz a -> a -> Fuzz a
delete = _delete

toList :: Fuzz a -> [(Integer, a)]
toList = _toList

fuzz :: (a -> a -> Bool) -> Integer -> Integer -> [(Integer, a)] -> Fuzz a
fuzz eq next count xs = Fuzz
  { _size = count
  , _insert = \x -> case listToMaybe (filter (eq x . snd) xs) of
      Nothing -> (next, fuzz eq (next + 1) (count + 1) ((next, x) : xs))
      Just (n, _) -> (n, fuzz eq next count xs)
  , _lookup = \n -> case listToMaybe (filter ((== n) . fst) xs) of
      Nothing -> Nothing
      Just (_, x) -> Just x
  , _elem = \x -> isJust $ listToMaybe (filter (eq x . snd) xs)
  , _delete = \x -> let (ys, zs) = partition (eq x . snd) xs
                    in  fuzz eq next (count - toInteger (length ys)) zs
  , _toList = xs
  }
```

15 hyperbolic/HalfPlane.hs

```
module HalfPlane
  ( Point(..)
  , eDist
```

```
    , dist
    , Geodesic(..)
    , fromPoints
    , fromPointAngle
    , atDist
    , angleAt
    , rotateAbout
    , midpoint
    , eMidpoint
    ) where

import Data.Fixed (mod')

eps :: Double
eps = 1e-12

data Point
  = Point{ pX, pY :: !Double }
  deriving Show

eDist :: Point -> Point -> Double
eDist p q =
  let dx = pX p - pX q
      dy = pY p - pY q
  in  sqrt (dx * dx + dy * dy)

dist :: Point -> Point -> Double
dist p q = case fromPoints p q of
  Line{}  -> abs (log (pY p / pY q))
  g@Arc{} ->
    let a = gCX g - gR g
        b = gCX g + gR g
        pa = sqrt ((pX p - a) * (pX p - a) + pY p * pY p)
        pb = sqrt ((pX p - b) * (pX p - b) + pY p * pY p)
        qa = sqrt ((pX q - a) * (pX q - a) + pY q * pY q)
        qb = sqrt ((pX q - b) * (pX q - b) + pY q * pY q)
    in  abs (log ((pa / pb) / (qa / qb)))

data Geodesic
  = Line{ gDir :: !Bool, gCX :: !Double }
  | Arc { gDir :: !Bool, gCX :: !Double, gR :: !Double }
  deriving Show

atParam :: Geodesic -> Double -> Point
atParam g@Line{} t = Point (gCX g) t
atParam g@Arc {} t = Point (gR g * cos (pi - t) + gCX g) (gR g * sin t)

paramAt :: Geodesic -> Point -> Double
paramAt   Line{} p = pY p
paramAt g@Arc {} p = pi - atan2 (pY p) (pX p - gCX g)

fromPoints :: Point -> Point -> Geodesic
fromPoints p q
  | abs (pX p - pX q) < eps =
          Line{ gCX = 0.5 * (pX p + pX q), gDir = pY p < pY q }
  | otherwise =
      let cx = 0.5 * (pX q * pX q + pY q * pY q - pX p * pX p - pY p * pY p) / (↙
```

```
          ↳ pX q - pX p)
          r = sqrt ((pX p - cx) * (pX p - cx) + pY p * pY p)
      in  Arc{ gCX = cx, gDir = pX p < pX q, gR = r }

fromPointAngle :: Point -> Double -> Geodesic
fromPointAngle p a
  | abs ((a `mod'` pi) - pi/2) < eps =
          Line{ gCX = pX p, gDir = cos a >= 0 }
  | otherwise =
      let b = a + pi/2
          co = cos b
          si = sin b
          r = abs (pY p / si)
          cx = -(pY p * co - pX p * si) / si
      in  Arc{ gCX = cx, gDir = cos a >= 0, gR = r }

atDist :: Geodesic -> Point -> Double -> Bool -> Point
atDist g@Line{} p d dir
  | dir /= gDir g = binarySearch g p d (0 + eps) pp  (<)
  | otherwise     = binarySearch g p d pp (pp * 100) (>)
  where pp = paramAt g p
atDist g@Arc {} p d dir
  | dir /= gDir g = binarySearch g p d (0 + eps) pp  (<)
  | otherwise     = binarySearch g p d pp (pi - eps) (>)
  where pp = paramAt g p

binarySearch :: Geodesic -> Point -> Double -> Double -> Double -> (Double -> ↙
    ↳ Double -> Bool) -> Point
binarySearch g p d lo hi cmp =
  let mid = 0.5 * (lo + hi)
      q = atParam g mid
      m = dist p q
  in  if hi - lo < eps
        then q
        else if m `cmp` d
               then binarySearch g p d lo mid cmp
               else binarySearch g p d mid hi cmp

angleAt :: Geodesic -> Point -> Double
angleAt g@Line{} _ = (if gDir g then (subtract pi) else id) (pi/2)
angleAt g@Arc {} p = (if gDir g then (pi +) else id) (atan2 (pY p) (pX p - gCX g↙
    ↳ ) + pi/2)

rotateAbout :: Point -> Double -> Point -> Point
rotateAbout p a q =
  let g = fromPoints p q
      b = angleAt g p
      h = fromPointAngle p (b + a)
      d = dist p q
  in  atDist h p d True

midpoint :: Point -> Point -> Point
midpoint p q = atDist (fromPoints p q) p (0.5 * dist p q) True

eMidpoint :: Point -> Point -> Point
eMidpoint p q = Point (0.5 * (pX p + pX q)) (0.5 * (pY p + pY q))
```

16 hyperbolic/Main.hs

```
{-# LANGUAGE TypeSynonymInstances #-}
module Main (main) where

import Control.Monad (when)
import Foreign hiding (rotate)
import Foreign.C (CUChar)
import Foreign.C.String
import Foreign.ForeignPtr (withForeignPtr)
import Graphics.Rendering.OpenGL.Raw
import Graphics.UI.GLUT hiding (rotate, translate, compileShader, RGBA, Triangle↙
    ↳ , Point)
import Data.Array.Repa.IO.DevIL
import Data.Array.Repa.Repr.ForeignPtr

import Paths_butterflies (getDataFileName)

import HalfPlane (Point(Point))
import Trapezium (toTriangles, neighbours, trapezium)
import Tesselate (tesselate)
import Colour (colour)
import Triangle (Triangle(Triangle), subtriangles)

swarm :: [Triangle]
swarm = concatMap subtriangles . concatMap toTriangles . colour $ tesselate (↙
    ↳ Point (-2) 0.01) (Point 2 4)

class GLPoke t where glPoke :: t -> Ptr t -> IO (Ptr t)
instance GLPoke t => GLPoke [t] where
  glPoke [] p = return p
  glPoke (x:xs) p = glPoke x (castPtr p) >>= glPoke xs . castPtr
instance GLPoke GLfloat where glPoke x p = poke p x >> return (advancePtr p 1)
instance GLPoke Double where
  glPoke x p = do
    p <- glPoke (realToFrac x :: GLfloat) (castPtr p)
    return (castPtr p)
instance GLPoke Point where
  glPoke (Point x y) p = do
    p <- glPoke x (castPtr p)
    p <- glPoke y p
    return (castPtr p)
instance GLPoke Triangle where
  glPoke (Triangle v0 v1 v2 t0 t1 t2 h0 h1 h2) p = do
    p <- glPoke [v0, t0] (castPtr p)
    p <- glPoke [h0, h1, h2] (castPtr p)
    p <- glPoke [v1, t1] (castPtr p)
    p <- glPoke [h0, h1, h2] (castPtr p)
    p <- glPoke [v2, t2] (castPtr p)
    p <- glPoke [h0, h1, h2] (castPtr p)
    return (castPtr p)

vert = unlines
  [ "#version 400 core"
  , "uniform mat4 mvp;"
  , "layout(location = 0) in vec2 vp0;"
  , "layout(location = 1) in vec2 tc0;"
```

```
    , "layout(location = 2) in vec3 hs0;"
    , "smooth out vec2 tc;"
    , "flat out vec3 hs;"
    , "void main() {"
    , "  vec4 vp = vec4(vp0, 0.0, 1.0) * mvp;"
    , "  gl_Position = vec4(vp.xy, 0.0, 1.0);"
    , "  tc = tc0;"
    , "  hs = hs0;"
    , "}"
    ]

frag = unlines
    [ "#version 400 core"
    , "uniform sampler2D tex;"
    , "uniform sampler1D pal;"
    , "smooth in vec2 tc;"
    , "flat in vec3 hs;"
    , "out layout(location = 0, index = 0) vec4 f;"
    , "void main() {"
    , "  vec4 c = vec4(0.0);"
    , "  for (int i = 0; i < 16; ++i) {"
    , "  for (int j = 0; j < 16; ++j) {"
    , "  vec2 t = tc + float(i)/16.0 * dFdx(tc) + float(j)/16.0 * dFdy(tc);"
    , "  vec4 w = texture(tex, t);"
    , "  if (w.a <= 0.5) {"
    , "    c += vec4(0.5, 0.5, 0.5, 1.0);"
    , "  } else if (w.r >= 0.5 && w.g >= 0.5 && w.b >= 0.5) {"
    , "    c += vec4(1.0, 1.0, 1.0, 1.0);"
    , "  } else if (w.r <= 0.5 && w.g <= 0.5 && w.b <= 0.5) {"
    , "    c += vec4(0.0, 0.0, 0.0, 1.0);"
    , "  } else if (w.r >= 0.5) {"
    , "    c += texture(pal, hs.r);"
    , "  } else if (w.g >= 0.5) {"
    , "    c += texture(pal, hs.g);"
    , "  } else if (w.b >= 0.5) {"
    , "    c += texture(pal, hs.b);"
    , "  } else {"
    , "    c = w;"
    , "  }}}"
    , "  f = c / 256.0;"
    , "}"
    ]

main = do
  _ <- getArgsAndInitialize
  _ <- createWindow "butterfly"
  loadTexture2D "butterfly.png"
  glActiveTexture gl_TEXTURE1
  loadTexture1D "palette.png"
  program <- compileProgram
  let count = length swarm * 3
      stride = (2 + 2 + 3) * 4
      bytes = count * fromIntegral stride
  allocaBytes bytes $ \p -> do
    glPoke swarm p
    vbo <- with 0 $ \q -> glGenBuffers 1 q >> peek q
    glBindBuffer gl_ARRAY_BUFFER vbo
```

```
      glBufferData gl_ARRAY_BUFFER (fromIntegral bytes) p gl_STATIC_DRAW
      vao <- with 0 $ \q -> glGenVertexArrays 1 q >> peek q
      glBindVertexArray vao
      glEnableClientState gl_VERTEX_ARRAY
      glVertexAttribPointer 0 2 gl_FLOAT (fromIntegral gl_FALSE) stride (plusPtr nullPtr (0 * 4))
      glVertexAttribPointer 1 2 gl_FLOAT (fromIntegral gl_FALSE) stride (plusPtr nullPtr (2 * 4))
      glVertexAttribPointer 2 3 gl_FLOAT (fromIntegral gl_FALSE) stride (plusPtr nullPtr (4 * 4))
      glEnableVertexAttribArray 0
      glEnableVertexAttribArray 1
      glEnableVertexAttribArray 2
    let l = -1.5
        r = 1.5
        t = 3
        b = 0
        n = -1
        f = 1
        ortho = [ 2 / (r - l), 0, 0, -(r + l) / (r - l)
                , 0, 2 / (t - b), 0, -(t + b) / (t - b)
                , 0, 0, 2 / (f - n), -(f + n) / (f - n)
                , 0, 0, 0, 1 ]
        ident = [ 1, 0, 0, 0, 0, 1, 0, -1, 0 ,0, 1, 0, 0, 0, 0, 1 ]
    glUseProgram program
    withArray ortho $ \p -> do
      loc <- withCString "mvp" $ glGetUniformLocation program
      glUniformMatrix4fv loc 1 (fromIntegral gl_FALSE) p
    withCString "tex" $ \p -> glGetUniformLocation program p >>= \loc -> glUniform1i loc 0
    withCString "pal" $ \p -> glGetUniformLocation program p >>= \loc -> glUniform1i loc 1
    glClearColor 0.5 0.5 0.5 1
    displayCallback $= do
      glClear gl_COLOR_BUFFER_BIT
      glDrawArrays gl_TRIANGLES 0 (fromIntegral count)
      swapBuffers
      reportErrors
    mainLoop

loadTexture2D f = do
  RGBA img <- getDataFileName f >>= runIL . readImage
  withForeignPtr (toForeignPtr img) $ \p -> do
    tex <- with 0 $ \q -> glGenTextures 1 q >> peek q
    glBindTexture gl_TEXTURE_2D tex
    glTexImage2D gl_TEXTURE_2D 0 (fromIntegral gl_RGBA) 1024 1024 0 gl_RGBA gl_UNSIGNED_BYTE p
    glTexParameteri gl_TEXTURE_2D gl_TEXTURE_MIN_FILTER (fromIntegral gl_LINEAR_MIPMAP_LINEAR)
    glTexParameteri gl_TEXTURE_2D gl_TEXTURE_MAG_FILTER (fromIntegral gl_LINEAR)
    glGenerateMipmap gl_TEXTURE_2D

loadTexture1D f = do
  RGBA img <- getDataFileName f >>= runIL . readImage
  withForeignPtr (toForeignPtr img) $ \p -> do
    tex <- with 0 $ \q -> glGenTextures 1 q >> peek q
    glBindTexture gl_TEXTURE_1D tex
```

```
    glTexImage1D gl_TEXTURE_1D 0 (fromIntegral gl_RGBA) 256 0 gl_RGBA ↙
        ↳ gl_UNSIGNED_BYTE p
    glTexParameteri gl_TEXTURE_1D gl_TEXTURE_MIN_FILTER (fromIntegral ↙
        ↳ gl_LINEAR_MIPMAP_LINEAR)
    glTexParameteri gl_TEXTURE_1D gl_TEXTURE_MAG_FILTER (fromIntegral gl_LINEAR)
    glGenerateMipmap gl_TEXTURE_1D

compileProgram = do
  program <- glCreateProgram
  compileShader program gl_VERTEX_SHADER vert
  compileShader program gl_FRAGMENT_SHADER frag
  glLinkProgram program
  debugProgram program
  return program

compileShader program t src = do
  shader <- glCreateShader t
  withCString src $ \srcp -> with srcp $ \srcpp -> glShaderSource shader 1 srcpp↙
      ↳ nullPtr
  glCompileShader shader
  glAttachShader program shader
  glDeleteShader shader

debugProgram program = do
  if program /= 0
    then do
      linked <- with 0 $ \p -> glGetProgramiv program gl_LINK_STATUS p >> peek p
      when (linked /= fromIntegral gl_TRUE) $ putStrLn "link failed"
      len <- with 0 $ \p -> glGetProgramiv program gl_INFO_LOG_LENGTH p >> peek ↙
          ↳ p
      allocaBytes (fromIntegral len + 1) $ \p -> glGetProgramInfoLog program len↙
          ↳ nullPtr p >> pokeByteOff p (fromIntegral len) (0 :: CUChar) >> ↙
          ↳ peekCString p >>= putStrLn
    else putStrLn "no program"
```

17 hyperbolic/Tesselate.hs

```
module Tesselate
  ( tesselate
  ) where

import HalfPlane (Point(Point), dist)
import Trapezium

elemBy :: (t -> t -> Bool) -> t -> [t] -> Bool
elemBy eq x [] = False
elemBy eq x (y:ys)
  | eq x y = True
  | otherwise = elemBy eq x ys

closure :: (t -> Bool) -> (t -> t -> Bool) -> (t -> [t]) -> t -> [t]
closure p e f x = closure' [] [x]
  where
    closure' _ [] = []
    closure' old xs = xs ++ closure' (old ++ new) new
      where
        new = [ y | y <- concatMap f xs, p y, not (elemBy e y old) ]
```

```
tesselate :: Point -> Point -> [Trapezium]
tesselate (Point lx ly) (Point hx hy) = closure visible approxEq neighbours ↙
    ↳ trapezium
  where
    approxEq s t = dist (origin s) (origin t) < 0.1
    visible = inBox . origin
    inBox (Point x y) = lx <= x && x <= hx && ly <= y && y <= hy
```

18 hyperbolic/Trapezium.hs

```
module Trapezium
  ( Trapezium(..)
  , trapezium
  , origin
  , neighbours
  , toTriangles
  ) where

import Data.List (partition)

import HalfPlane
import Triangle

data Trapezium = Trapezium{ bTL, bTM, bTR, bBR, bBL, bFR, bFL :: Point, bSpots, ↙
    ↳ bWings, bBody :: Int }
  deriving Show

trapezium :: Trapezium
trapezium = Trapezium tl tm tr br bl fr fl (-1) (-1) (-1)
  where
    tl = Point 0 1
    t  = fromPointAngle tl ( pi / 7)
    l  = fromPointAngle tl (-pi / 7)
    d  = fromPointAngle tl 0
    tm = atDist t tl (1 * s / 2) True
    tr = atDist t tl (2 * s / 3) True
    fr = atDist t tl (3 * s / 3) True
    fl = atDist l tl (3 * s / 3) True
    bl = atDist l tl (    s / 3) True
    br = atDist d tl r True
    s = acosh ((cos (2 * pi / 7) + cos (2 * pi / 7) ^ two) / sin (2 * pi / 7) ^ ↙
        ↳ two)
    r = asinh (sinh s * sin (pi / 7) / sin (2 * pi / 3))
    two :: Int
    two = 2

origin :: Trapezium -> Point
origin = bBL

neighbours :: Trapezium -> [Trapezium]
neighbours z =
  [ (rotateAbout' (bTL z) (2 * pi / 7) z){ bTL = bTL z }
  , (rotateAbout' (bTM z)       pi     z){ bTM = bTM z }
  , (rotateAbout' (bBR z) (2 * pi / 3) z){ bBR = bBR z }
  ]
```

```
rotateAbout' :: Point -> Double -> Trapezium -> Trapezium
rotateAbout' p a b = b
  { bTL = rotateAbout p a (bTL b)
  , bTM = rotateAbout p a (bTM b)
  , bTR = rotateAbout p a (bTR b)
  , bBR = rotateAbout p a (bBR b)
  , bBL = rotateAbout p a (bBL b)
  , bFR = rotateAbout p a (bFR b)
  , bFL = rotateAbout p a (bFL b)
  }

toTriangles :: Trapezium -> [Triangle]
toTriangles (Trapezium tl _ tr br bl _ _ c0 c1 c2) =
  [ Triangle tl tm bl (t 0   1) (t 0.5  1) (t 0.25 0) h0 h1 h2
  , Triangle tm tr br (t 0.5 1) (t 1    1) (t 0.75 0) h0 h1 h2
  , Triangle tm br bl (t 0.5 1) (t 0.75 0) (t 0.25 0) h0 h1 h2
  ]
  where
    h0 = fromIntegral c0 / 24
    h1 = fromIntegral c1 / 24
    h2 = fromIntegral c2 / 24
    tm = midpoint tl tr
    t = Point
```

19 hyperbolic/Triangle.hs

```
module Triangle
  ( Triangle(..)
  , subtriangles
  ) where

import HalfPlane

data Triangle = Triangle{ tV0, tV1, tV2, tT0, tT1, tT2 :: Point, tH0, tH1, tH2 ↙
    ↳ :: Double }
  deriving Show

subtriangles :: Triangle -> [Triangle]
subtriangles (Triangle v0 v1 v2 t0 t1 t2 h0 h1 h2) =
  [ Triangle v0  v01 v02 t0  t01 t02 h0 h1 h2
  , Triangle v1  v01 v12 t1  t01 t12 h0 h1 h2
  , Triangle v2  v02 v12 t2  t02 t12 h0 h1 h2
  , Triangle v01 v02 v12 t01 t02 t12 h0 h1 h2
  ]
  where
    v01 = midpoint v0 v1
    v02 = midpoint v0 v2
    v12 = midpoint v1 v2
    t01 = eMidpoint t0 t1
    t02 = eMidpoint t0 t2
    t12 = eMidpoint t1 t2
```

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Version 3, 29 June 2007

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```
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```

21 NEWS

```
0.2.0.0 hyperbolic {7,3}/24 tiling
0.1.0.1 thicker outlines on butterfly tile
0.1.0.0 initial version {6,3}/4 butterfly tiling
```

22 OpenGLRaw21.patch

```
diff -ruw OpenGLRaw21-1.2.0.1/OpenGLRaw21.cabal OpenGLRaw21-1.3.0.0/OpenGLRaw21.↙
    ↳ cabal
```

```
--- OpenGLRaw21-1.2.0.1/OpenGLRaw21.cabal       2013-01-10 18:11:57.000000000 ↙
    ↘ +0000
+++ OpenGLRaw21-1.3.0.0/OpenGLRaw21.cabal       2013-01-10 18:12:47.000000000 ↙
    ↘ +0000
@@ -1,5 +1,5 @@
 Name:                OpenGLRaw21
-Version:             1.2.0.1
+Version:             1.3.0.0
 Synopsis:            The intersection of OpenGL 2.1 and OpenGL 3.1 Core
 Description:         This package simply reexports a subset of the
                      parts of OpenGLRaw which are compatible with
@@ -18,7 +18,7 @@
 Cabal-version:       >=1.6

 Library
-  Build-depends:     OpenGLRaw == 1.1.* || == 1.2.*
+  Build-depends:     OpenGLRaw == 1.1.* || == 1.2.* || == 1.3.*
   Extensions:        NoImplicitPrelude
   Exposed-modules:   Graphics.Rendering.OpenGL.Raw.Core21
   GHC-options:       -Wall -fwarn-tabs
```

23 README

```
butterflies -- butterfly tilings
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Example usage:

  #!/bin/bash
  for s in $(seq 2 5)
  do
    for p in $(seq 0 $((s/2)))
    do
      q=$((s-p))
      butterflies-flat $q $p -geometry 1024x1024
    done
  done
```

24 Setup.hs

```
import Distribution.Simple
main = defaultMain
```