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1 benchmark.cc

```
#include <iostream>
     #define CL_TARGET_OPENCL_VERSION 120
     \#define __CL_ENABLE_EXCEPTIONS
     #include <CL/cl.hpp>
     #include "fixed-point.c"
     const char fixed_point_cl[] = {
     #include "fixed-point.c.h"
10
     , 0
     };
     int main(int argc, char** argv)
        if (argc < 10)
15
          std::fprintf
               "usage: %s width height cx cy r maxiters use_fixed use_cl use_bigendian ∠
                 \hookrightarrow > out.ppm\n"
             , argv [0]
20
             );
          return 1;
        int subframe = 1;
25
        int width = atoi(argv[1]);
        int height = atoi(argv[2]);
        double r0 = atof(argv[5]);
        int prec = -\log 2(2 * r0 / \text{height});
        \label{eq:charge_bound} \verb|int| b = \verb|sizeof(fixed_word_t)| * CHAR\_BIT;
        int LIMBS = (prec + 8 + b - 1) / b;
30
        fixed\_word\_t cx0[1 + LIMBS], cy0[1 + LIMBS];
        \label{eq:fixed_from_string} \texttt{fixed\_from\_string} \, (\, \texttt{cx0} \, , \, \, \, 1 \, , \, \, \texttt{LIMBS}, \, \, \, \texttt{argv} \, [\, 3 \, ] \, ) \, ;
        fixed_from_string(cy0, 1, LIMBS, argv[4]);
        int maxiters = atoi(argv[6]);
```

fixed-point benchmark.cc

```
35
       bool use_fixed = atoi(argv[7]);
       bool use_cl = atoi(argv[8]);
       bool use_bigendian = atoi(argv[9]);
       if (use_fixed)
       {
         \mathtt{std} :: \mathtt{cerr} << " \, \mathtt{fixed} \  \, \mathtt{format} \  \, \mathtt{Q1."} \  \, << \, \mathtt{LIMBS} << \, \, \mathtt{std} :: \mathtt{endl} \, ;
40
       size_t bytes = (size_t) height * width * 3;
       unsigned char* results = new unsigned char[bytes];
45
       if (use_cl)
       {
         // OpenCL renderers
         try {
            std::vector<cl::Platform> platforms;
            cl::Platform::get(&platforms);
50
            std::vector<cl::Device> platformDevices;
            platforms \ [0\,]. \ getDevices (CL\_DEVICE\_TYPE\_GPU, \ \&platformDevices);
            cl::Context context(platformDevices);
            auto contextDevices = context.getInfo<CL_CONTEXT_DEVICES>();
55
            auto device = contextDevices[0];
            \mathtt{std} :: \mathtt{cerr} << \mathtt{``Using} \ `` << \mathtt{device} . \mathtt{getInfo} <\!\! \mathtt{CL.DEVICE\_NAME} >\!\! () << \mathtt{std} :: \mathtt{endl} ;
            cl::Program::Sources source
               , std::make_pair(fixed_point_cl, sizeof(fixed_point_cl) - 1)
60
            cl::Program program(context, source);
              bool device_big_endian = use_bigendian;
    #ifdef BIG_ENDIAN
65
              bool host_big_endian = true;
    #else
              bool host_big_endian = false;
    #endif
              char options [100];
70
              std::snprintf
                 (options, 100
                  "-D LIMBS=%d%s%s"
                 , LIMBS
                 , device_big_endian ? " -D BIG_ENDIAN=1" : ""
                 , device_big_endian != host_big_endian ? " -D SWAP_ENDIAN=1" : ""
75
                );
              program.build(contextDevices, options);
            } catch (cl::Error &e) {
              auto buildlog = program.getBuildInfo<CLPROGRAM.BUILDLOG>(device);
80
              std::cerr << buildlog << std::endl;
              return 1;
            cl::CommandQueue queue(context, device, CL_QUEUE_PROFILING_ENABLE);
            cl_int err = CL_SUCCESS;
            cl::Buffer output(context, CLMEM_WRITE_ONLY, bytes, nullptr, &err);
85
            cl::NDRange offset(0, 0);
            cl::NDRange global_size(height, width);
            if (use_fixed)
              cl::Buffer cx0_buffer
90
                 ( context
```

fixed-point benchmark.cc

```
, CL_MEM_READ_ONLY | CL_MEM_COPY_HOST_PTR
                 (1 + LIMBS) * size of (fixed_word_t)
                , cx0
95
                , &err
               );
             cl::Buffer cy0_buffer
               ( context
                , CLMEM_READ_ONLY | CL_MEM_COPY_HOST_PTR
100
               (1 + LIMBS) * sizeof(fixed_word_t)
               , cy0
                 &err
               );
             cl::Kernel mandelbrot(program, "mandelbrot_fixed");
             mandelbrot.setArg(0, output);
105
             mandelbrot.setArg(1, subframe);
             mandelbrot.setArg(2, height);
             mandelbrot.setArg(3, width);
             mandelbrot.setArg(4, cx0_buffer);
110
             mandelbrot.setArg(5, cy0_buffer);
             mandelbrot.setArg(6, r0);
             mandelbrot.setArg(7, maxiters);
             queue.enqueueNDRangeKernel(mandelbrot, offset, global_size);
           }
115
           else
             double cx0_double = fixed_to_double(cx0, 1, LIMBS);
             double cy0_double = fixed_to_double(cy0, 1, LIMBS);
             cl::Kernel mandelbrot(program, "mandelbrot_double");
120
             mandelbrot.setArg(0, output);
             mandelbrot.setArg(1, subframe);
             mandelbrot.setArg(2, height);
             mandelbrot.setArg(3, width);
             mandelbrot.setArg(4, cx0_double);
             mandelbrot.setArg(5, cy0_double);
125
             mandelbrot.setArg(6, r0);
             mandelbrot.setArg(7, maxiters);
             queue.enqueueNDRangeKernel(mandelbrot, offset, global_size);
           }
130
           queue.enqueueBarrierWithWaitList();
           cl::Event readDoneEvent;
           queue.enqueueReadBuffer
             ( output
              , CL_FALSE
             , 0
135
             , bytes
               results
               nullptr
               &readDoneEvent
140
           std::vector<cl::Event> readWaitList;
           {\tt readWaitList.push\_back(readDoneEvent);}
           cl::Event::waitForEvents(readWaitList);
          catch (cl::Error &e) {
           std::cerr << e.what() << ": error code " << e.err() << std::endl;
145
           return 1;
         }
       }
```

```
else
150
         // CPU renderers
         if (use\_fixed)
            mandelbrot\_fixed
              ( results
155
              , subframe
              , height
                width
               cx0
              , cy0
160
              , r0
              , maxiters
              , LIMBS
              );
         }
165
         else
            double cx = fixed_to_double(cx0, 1, LIMBS);
            double cy = fixed_to_double(cy0, 1, LIMBS);
170
            mandelbrot_double
              ( results
              , subframe
              , height
              , width
175
              , cx
              , су
              , r0
              , maxiters
              );
180
         }
       }
       // output image
       std::fprintf(stdout, "P6\n%d %d\n\# cx = ", width, height);
       fixed_dump(stdout, cx0, 1, LIMBS);
       std::fprintf(stdout, "\n# cy = ");
185
       fixed_dump(stdout, cy0, 1, LIMBS);
       std::fprintf
           stdout
            "\n# r = %.18e\n# maxiters = %d\n# use_fixed = %d\n# use_cl = %d\n255\n"
           r0
190
           maxiters
          , use_fixed
          , use_cl
195
       std::fwrite(results, bytes, 1, stdout);
       return 0;
     }
```

2 fixed-point.c

```
/* Each number is stored as pointer to limbs, with counts of integer and fractional parts.
```

5 #ifdef BIG_ENDIAN

```
The limbs are stored in big-endian order (each limb
    is stored in native-endian order):
         -(i-1) ... -2 -1 0 . 1 2 .. f
10
    The represented value is:
    \ \sum_{n=0}^{i+f} p \left[ \frac{n \cdot n}{n \cdot n} \right] b^{i-1} - n 
15
    #else
    The limbs are stored in little-endian order (each limb
    is stored in native-endian order):
20
         f \dots 2 1 \dots 0 -1 -2 \dots -(i-1)
    The represented value is:
25
    \ \sum_{n=0}^{i} i + f} p\left\[n\right\] b^{n - f} $$
    #endif
    where b = 2^{number} of bits in a word.
30
    */
    #ifdef __OPENCL_VERSION__
    // OpenCL
35
    #pragma OPENCLEXTENSION cl_khr_fp64 : enable
    #ifndef LIMBS
    #error LIMBS not defined
    #endif
    // limb types: wordword should have twice the bits of word
    typedef uint fixed_word_t;
    typedef ulong fixed_wordword_t;
45
    #define static
    #define inline
    #define assert(x) do{} while(0)
50
    #else
    // C99 / C11 / C++11+VLA / ...
    #include <assert.h>
55
    #include <float.h>
    #include <limits.h>
    #include <math.h>
    #include <stdbool.h>
    #include <stdint.h>
    #include <stdio.h>
    #include <stdlib.h>
```

```
// only used for string->fixed conversion
65
     #include <mpfr.h>
     // limb types: wordword should have twice the bits of word
     typedef uint32_t fixed_word_t;
     typedef uint64_t fixed_wordword_t;
70
     #define __kernel
     #define __global
     #define __constant const
 75
     #endif
     static inline
     fixed_word_t fixed_msb(const fixed_wordword_t ww)
       return ww >> (sizeof(fixed_word_t) * CHAR_BIT);
80
     static inline
     fixed_word_t fixed_lsb(const fixed_wordword_t ww)
85
       return ww;
     static inline
     fixed_wordword_t fixed_word_mul(fixed_word_t a, fixed_word_t b)
90
       return (fixed_wordword_t) a * (fixed_wordword_t) b;
     }
     static inline
     bool fixed_lt_zero
     ( const fixed_word_t *a, int ai, int af
       if (ai == 0 && af == 0)
100
         return false;
     #ifdef BIG_ENDIAN
       int ix = 0;
105
     #else
       int ix = ai + af - 1;
       return !!(a[ix] & ((fixed_word_t) 1 << (sizeof(fixed_word_t) * CHAR_BIT - 1))) \( \alpha \)
110
     }
     static inline
     fixed_word_t fixed_read_be
     ( const fixed_word_t *a, int ai, int af
115
       int aw
       if (ai == 0 && af == 0)
```

```
{
          return (fixed_word_t) 0;
120
       if (aw < -(ai - 1))
          // sign extend
          if (fixed_lt_zero(a, ai, af))
125
           return ~ (fixed_word_t) 0;
         _{\rm else}
130
          {
                     (fixed_word_t) 0;
           return
       if (aw >= ai + af)
135
         // trailing 0
         return (fixed_word_t) 0;
       int ix = (ai - 1) + aw;
140
       return a[ix];
     static inline
     fixed_word_t fixed_read_le
     ( const fixed_word_t *a, int ai, int af
145
       int aw
       if (ai = 0 \&\& af = 0)
150
         return (fixed_word_t) 0;
       if (aw < -(ai - 1))
155
          // sign extend
          if (fixed_lt_zero(a, ai, af))
           return ~ (fixed_word_t) 0;
160
          else
                     (fixed_word_t) 0;
            return
165
       if (aw >= ai + af)
          // trailing 0
          return (fixed_word_t) 0;
       int ix = af - aw;
170
       return a[ix];
     }
     static inline
     void fixed_write_be
175
```

```
( fixed_word_t *a, int ai, int af
        int aw
        fixed_word_t w
180
        assert(-(ai - 1) \le aw);
        assert (aw <= af);
        int ix = (ai - 1) + aw;
        a[ix] = w;
185
      static inline
      void fixed_write_le
      ( fixed_word_t *a, int ai, int af
190
      , int aw
        fixed\_word\_t w
        assert(-(ai - 1) \le aw);
195
        assert (aw <= af);
        int ix = af - aw;
        a[ix] = w;
     #ifdef BIG_ENDIAN
200
     #define fixed_read
                            fixed_read_be
     #define fixed_write fixed_write_be
     #else
     #define fixed_read
                            fixed_read_le
205
     #define fixed_write fixed_write_le
     #endif
      static inline
      fixed_word_t fixed_to_word
210
      ( const fixed_word_t *a, int ai, int af
        return fixed_read(a, ai, af, 0);
215
      double fixed_to_double
      ( const fixed_word_t *a, int ai, int af
        if (fixed_lt_zero(a, ai, af))
220
        {
          double o = 0;
          for (int aw = -(ai - 1); aw \le af; ++aw)
            \label{eq:fixed_word_t} \mbox{fixed_word_t} \ \mbox{limb} \ = \ \mbox{fixed_read} \left( \mbox{a} \mbox{, ai, af, aw} \right);
225
            o += ldexp((double) ~limb, -aw * sizeof(fixed_word_t) * CHAR_BIT);
          return -o;
        }
230
        else
        {
          double o = 0;
```

```
for (int aw = -(ai - 1); aw \le af; ++aw)
            fixed_word_t limb = fixed_read(a, ai, af, aw);
235
            o += ldexp((double) limb, -aw * sizeof(fixed_word_t) * CHAR_BIT);
          return o;
240
     }
     void fixed_zero
     ( fixed_word_t *a, int ai, int af
245
        for (int ix = 0; ix < ai + af; ++ix)
         a[ix] = 0;
250
     }
     void fixed_copy
              \label{eq:fixed_word_t *o, int oi, int of} fixed_word_t *o, int oi, int of
       const fixed_word_t *a, int ai, int af
255
        if (o = a \&\& oi = ai \&\& of = af)
          return;
260
        for (int ow = -(oi - 1); ow \leq of; ++ow)
          fixed_word_t w = fixed_read(a, ai, af, ow);
          fixed_write(o, oi, of, ow, w);
       }
265
       return;
     }
     void fixed_shift
              fixed_word_t *o, int oi, int of
270
       const fixed_word_t *a, int ai, int af
       int b
275
        if (b = 0)
          fixed_copy(o, oi, of, a, ai, af);
          return;
        if (b > 0)
280
          // left to right, to support o == a
          int extract = b % (sizeof(fixed_word_t) * CHAR_BIT);
          int shift = b / (sizeof(fixed_word_t) * CHAR_BIT);
          int ow = -(oi - 1);
285
          fixed\_wordword\_t w = 0;
         w <<= sizeof(fixed_word_t) * CHAR_BIT;
         w = fixed_read(a, ai, af, ow + shift);
          for (; ow \le of; ++ow)
```

```
290
           w <<= sizeof(fixed_word_t) * CHAR_BIT;</pre>
           295
         return;
       if (b < 0)
         // right to left, to support o == a
300
         b = -b;
         int extract = b % (sizeof(fixed_word_t) * CHAR_BIT);
         int shift = b / (sizeof(fixed_word_t) * CHAR_BIT);
         int ow = of;
         fixed\_wordword\_t w = 0;
         w >>= sizeof(fixed_word_t) * CHAR_BIT;
305
         w |= (fixed_wordword_t) fixed_read(a, ai, af, ow - shift)
            << (sizeof(fixed_word_t) * CHAR_BIT);</pre>
         for (; ow >= -(oi - 1); --ow)
310
           w >>= sizeof(fixed_word_t) * CHAR_BIT;
           w |= (fixed_wordword_t) fixed_read(a, ai, af, ow - (shift + 1))
               << (sizeof(fixed_word_t) * CHAR_BIT);</pre>
           fixed_write(o, oi, of, ow, fixed_lsb(w >> extract));
315
         return;
     }
     fixed_word_t fixed_neg
             fixed_word_t *o, int oi, int of
320
       const fixed_word_t *a, int ai, int af
       fixed\_word\_t \ carry = 1;
325
       for (int ow = of; ow \rightarrow = -(oi - 1); --ow)
         fixed_word_t in_a = fixed_read(a, ai, af, ow);
         fixed\_word\_t not\_a = \tilde{in\_a};
         fixed_word_t a_result = not_a + carry;
         carry = a_result < not_a ? 1 : 0;
330
         fixed_write(o, oi, of, ow, a_result);
       }
       return carry;
335
     void fixed_from_word
     ( fixed_word_t *a, int ai, int af
       fixed_word_t x
340
       fixed_zero(a, ai, af);
       fixed_write(a, ai, af, 0, x);
     }
     void fixed_from_double
345
     ( fixed_word_t *a, int ai, int af
```

```
double x
        // FIXME may be inaccurate if 2 > af
350
        assert(FLT\_RADIX == 2);
        if (x = 0)
          fixed_zero(a, ai, af);
355
          return;
        bool negate = x < 0;
        if (negate) x = -x;
        int e = 0;
360
        x = frexp(x, \&e);
        // [0.5 .. 1), original value is x * 2^e
        for (int aw = -(ai - 1); aw <= 0; ++ aw)
          fixed_write(a, ai, af, aw, 0);
365
        for (int aw = 1; aw \le af; ++aw)
          x = ldexp(x, sizeof(fixed_word_t) * CHAR_BIT);
          fixed_word_t w = floor(x);
370
          x = (double) w;
          fixed_write(a, ai, af, aw, w);
          // also writes trailing 0s
        fixed_shift(a, ai, af, a, ai, af, e);
        if (negate)
375
          fixed_neg(a, ai, af, a, ai, af);
        }
        return;
380
      fixed_word_t fixed_add
              fixed_word_t *o, int oi, int of
       const\ fixed\_word\_t\ *a\,,\ int\ ai\,,\ int\ af
385
        const fixed_word_t *b, int bi, int bf
        fixed\_word\_t \ carry = 0;
        for (int ow = of; ow \rightarrow = -(oi - 1); --ow)
390
          \label{eq:fixed_word_time} fixed\_word\_t \ in\_a = fixed\_read\left(a\,,\ ai\,,\ af\,,\ ow\right);
          fixed_word_t in_b = fixed_read(b, bi, bf, ow);
          fixed_word_t a_result = in_a + carry;
          fixed_word_t a_carry = a_result < in_a ? 1 : 0;
          fixed_word_t b_result = in_b + a_result;
395
          fixed_word_t b_carry = b_result < in_b ? 1 : 0;</pre>
          carry = a_carry + b_carry;
          fixed_write(o, oi, of, ow, b_result);
        }
400
        return carry;
      fixed_word_t fixed_sub
```

```
fixed_word_t *o, int oi, int of
       const fixed_word_t *a, int ai, int af
const fixed_word_t *b, int bi, int bf
405
        fixed\_word\_t borrow = 0;
        \  \, \text{for (int ow} \, = \, \text{of; ow} \, > = \, -(\, \text{oi} \, - \, 1) \, ; \, \, - \text{-ow}) \,
410
          fixed_word_t in_a = fixed_read(a, ai, af, ow);
          fixed_word_t in_b = fixed_read(b, bi, bf, ow);
          fixed_word_t a_result = in_a - borrow;
          \label{eq:fixed_word_taborrow} \mbox{fixed_word_t a_borrow} = \mbox{borrow} > \mbox{in_a} \ ? \ 1 \ : \ 0;
415
          fixed_word_t b_result = a_result - in_b;
          fixed\_word\_t b\_borrow = in\_b > a\_result ? 1 : 0;
          borrow = a_borrow + b_borrow;
          fixed_write(o, oi, of, ow, b_result);
420
        return borrow;
      }
      fixed_word_t fixed_mul
425
               fixed_word_t *o, int oi, int of
       const fixed_word_t *a, int ai, int af
        const fixed_word_t *b, int bi, int bf
        fixed\_wordword\_t accum = 0;
430
        fixed_wordword_t carry = 0;
        for (int ow = of; ow \Rightarrow -(oi - 1); --ow)
          accum = fixed_lsb(carry);
435
          carry = fixed_msb(carry);
          if (ow == of)
          {
             // sum fixed-msb over aw + bw = ow + 1, aw <= af, bw <= bf
             for (int aw = ow + 1 - bf; aw \le af; ++aw)
440
               int bw = ow + 1 - aw;
               fixed\_word\_t in\_a = fixed\_read(a, ai, af, aw);
               fixed_word_t in_b = fixed_read(b, bi, bf, bw);
               fixed_wordword_t ab = fixed_word_mul(in_a, in_b);
445
               accum += fixed_msb(ab);
          // sum fixed_lsb over aw + bw = ow, aw <= af, bw <= bf
          for (int aw = ow - bf; aw \le af; ++aw)
450
             int bw = ow - aw;
             fixed_word_t in_a = fixed_read(a, ai, af, aw);
             fixed\_word\_t in\_b = fixed\_read(b, bi, bf, bw);
             fixed_wordword_t ab = fixed_word_mul(in_a, in_b);
            accum += fixed_lsb(ab);
455
             carry += fixed_msb(ab);
          fixed_write(o, oi, of, ow, fixed_lsb(accum));
          carry += fixed_msb(accum);
460
        }
```

fixed-point fixed-point.c

```
return fixed_lsb(carry);
     fixed_word_t fixed_sqr
              fixed_word_t *o, int oi, int of
465
       const\ fixed\_word\_t\ *a\,,\ int\ ai\,,\ int\ af
       fixed_wordword_t accum = 0;
       fixed_wordword_t carry = 0;
470
       for (int ow = of; ow \Rightarrow -(oi - 1); --ow)
         accum = fixed_lsb(carry);
         carry = fixed_msb(carry);
475
         if (ow == of)
           // sum fixed_msb over aw + bw = ow + 1, aw <= af, bw <= bf, exploiting 2
               \  \, \text{for (int aw = ow + 1 - af; aw <= af; ++aw)} \,
480
              int bw = ow + 1 - aw;
              if (aw \le bw)
              {
                fixed_word_t in_a = fixed_read(a, ai, af, aw);
                fixed_word_t in_b = aw == bw ? in_a : fixed_read(a, ai, af, bw);
                fixed_wordword_t ab = fixed_word_mul(in_a, in_b);
485
                if (aw != bw)
                  accum += (fixed_wordword_t) fixed_msb(ab) << 1;
                }
                else
490
                {
                  accum += fixed_msb(ab);
              }
           }
495
         // sum fixed_lsb over aw + bw = ow, aw <= af, bw <= bf, exploiting symmetry
         for (int aw = ow - af; aw \le af; ++aw)
            int bw = ow - aw;
500
            if (aw \le bw)
              fixed_word_t in_a = fixed_read(a, ai, af, aw);
              fixed_word_t in_b = aw == bw ? in_a : fixed_read(a, ai, af, bw);
505
              fixed_wordword_t ab = fixed_word_mul(in_a, in_b);
              if (aw != bw)
              {
                accum += (fixed_wordword_t) fixed_lsb(ab) << 1;
                carry += (fixed_wordword_t) fixed_msb(ab) << 1;</pre>
              }
510
              else
              {
                accum += fixed_lsb(ab);
                carry += fixed_msb(ab);
515
           }
```

```
fixed_write(o, oi, of, ow, fixed_lsb(accum));
           carry += fixed_msb(accum);
520
        return fixed_lsb(carry);
      static inline
525
      double double_sqr(double x)
        return x * x;
      }
530
      double mandelbrot_double_core(double cx, double cy, int maxiters)
        double zx = 0;
        double zy = 0;
        for (int n = 0; n < maxiters; ++n)
535
           double zx2 = double sqr(zx);
           double zy2 = double sqr(zy);
           double z2 = zx2 + zy2;
           if (z2 > = 256)
540
             return n + 1 - \log 2 (0.5 * \log (z2));
           double zxy = zx * zy;
          zx = zx2 - zy2 + cx;
          zy = 2 * zxy + cy;
545
        }
        return 1.0 / 0.0; // infinity for interior
      }
550
      local memory usage in bytes:
           ((i + f) * 5 + 3) * sizeof(word)
555
      */
      double mandelbrot_fixed_core
      ( const fixed_word_t *cx
      , const\ fixed\_word\_t\ *cy
      , int maxiters
     #ifdef __OPENCL_VERSION__
560
      // no VLA, use global #define
     #else
      , int LIMBS
     #endif
565
     #define i 1
     #define f LIMBS
     \#define w (i + f)
570
        \label{eq:fixed_word_t} \text{fixed_word_t} \ \ \text{zx}\,[w] \ , \ \ \text{zy}\,[w] \ , \ \ \text{zx2}\,[w] \ , \ \ \text{zy2}\,[w] \ , \ \ \text{zxy}\,[w] \ , \ \ \text{z2}\,[1+2];
        fixed_zero(zx, i, f);
        fixed_zero(zy, i, f);
        for (int n = 0; n < maxiters; ++n)
```

```
{
575
             \label{eq:fixed_sqr} \mbox{fixed_sqr} \left( \, zx2 \; , \  \  i \; , \  \  f \; , \; \; zx \; , \; \; i \; , \; \; f \; \right) \; ;
             \begin{array}{l} \mbox{fixed\_sqr} \left( \, zy2 \,\, , \  \, i \,\, , \,\, f \,\, , \,\, zy \,\, , \,\, i \,\, , \,\, f \,\, \right) \,; \\ \mbox{fixed\_add} \left( \, z2 \,\, , \,\, 1 \,\, , \,\, 2 \,\, , \,\, zx2 \,\, , \,\, i \,\, , \,\, f \,\, , \,\, zy2 \,\, , \,\, i \,\, , \,\, f \,\, \right) \,; \end{array}
             if (fixed_to_word(z2, 1, 2) >= 256)
                double d = fixed_to_double(z2, 1, 2);
580
                return n + 1 - \log 2(0.5 * \log(d));
             fixed_mul(zxy, i, f, zx, i, f, zy, i, f);
             fixed\_sub(zx, i, f, zx2, i, f, zy2, i, f);
             \label{eq:fixed_add(zx, i, f, zx, i, f, cx, i, f);} fixed_add(zx, i, f, zx, i, f, cx, i, f);
585
             fixed_shift(zy, i, f, zxy, i, f, 1);
             fixed_add(zy, i, f, zy, i, f, cy, i, f);
          }
          return 1.0 / 0.0; // infinity for interior
      #undef i
590
      #undef f
      #undef w
595
       fixed_word_t hash_burtle_9 (fixed_word_t a)
          // FIXME assumes 32 bit word
          a = (a+0x7ed55d16u) + (a<<12u);
          a = (a^0xc761c23cu)^{-1} (a>>19u);
600
          a = (a+0x165667b1u) + (a<<5u);
          a = (a+0xd3a2646cu) ^
                                        (a << 9u);
          a = (a+0xfd7046c5u) + (a<<3u);
          a = (a^0xb55a4f09u)^{-1} (a>>16u);
          return a;
605
       }
       double uniform01 (int seed)
          fixed_word_t r = hash_burtle_9 (seed);
610
          return (double)(r) / ((double)(fixed_word_t)(-1) + 1);
       __kernel
       void mandelbrot_double
615
       ( __global unsigned char* output
         int subframe
       , int height
       , int width
        double cx0
620
        double cy0
          double r0
          int maxiters
      #ifdef __OPENCL_VERSION__
625
        int y = get_global_id(0);
        int x = get_global_id(1);
        #pragma omp parallel for
630
        for (int y = 0; y < height; ++y)
```

```
for (int x = 0; x < width; ++x)
     #endif
       int k = (y * width + x) * 3;
       int seed = subframe * width * height + y * width + x;
635
       double dx = 2 * r0 * ((x + uniform01(2 * seed + 0)) / width - 0.5) * width / <math>2
           ↳ height;
       double dy = 2 * r0 * ((height - (y + uniform01(2 * seed + 1)))) / height - 0.5) <math>\checkmark
           >;
       double mu = mandelbrot_double_core(cx0 + dx, cy0 + dy, maxiters);
       if (mu = -1.0 / 0.0)
640
         output[k + 0] = 0;
         output[k + 1] = 0;
         output[k + 2] = 0;
       }
645
       else
       {
         output [k + 0] = 255 * (0.5 - 0.5 * cos(mu / 1));
         output[k + 1] = 255 * (0.5 - 0.5 * cos(mu / 10));
         output [k + 2] = 255 * (0.5 - 0.5 * cos(mu / 100));
650
       }
      }
     __kernel
     void mandelbrot_fixed
655
     ( __global unsigned char* output
     , int subframe
     , int height
     , int width
     , __constant fixed_word_t *cx0
660
       __constant fixed_word_t *cy0
     , double r0
      int maxiters
     #ifdef __OPENCL_VERSION__
665
     // no VLA, use global #define
     #else
     , int LIMBS
     #endif
670
     #ifdef __OPENCL_VERSION__
      int y = get_global_id(0);
      int x = get_global_id(1);
     #else
675
      #pragma omp parallel for
      for (int y = 0; y < height; +++y)
      for (int x = 0; x < width; ++x)
     #endif
680
       int k = (y * width + x) * 3;
       int seed = subframe * width * height + y * width + x;
       double dx = 2 * r0 * ((x + uniform01(2 * seed + 0)) / width - 0.5) * width / \varrangle
       double dy = 2 * r0 * ((height - (y + uniform01(2 * seed + 1)))) / height - 0.5) <math>\angle
           \;
```

```
\label{eq:cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_cond_to_co
685
                        \label{eq:fixed_from_double(cx, 1, LIMBS, dx);} \text{ fixed\_from\_double(cx, 1, LIMBS, dx);}
                        fixed_from_double(cy, 1, LIMBS, dy);
                              fixed_word_t t[1+LIMBS];
                              for (int i = 0; i \le LIMBS; ++i)
690
                              {
                #ifdef SWAP_ENDIAN
                                    t[i] = cx0[LIMBS - i];
                #else
                                     t[i] = cx0[i];
                #endif
695
                              fixed_add(cx, 1, LIMBS, cx, 1, LIMBS, t, 1, LIMBS);
                              for (int i = 0; i \le LIMBS; ++i)
                #ifdef SWAP_ENDIAN
700
                                   t[i] = cy0[LIMBS - i];
                #else
                                    t[i] = cy0[i];
                #endif
705
                              fixed_add(cy, 1, LIMBS, cy, 1, LIMBS, t, 1, LIMBS);
                        double mu = mandelbrot_fixed_core
                              (cx
                              , су
710
                               , maxiters
                #ifdef __OPENCL_VERSION__
                             // no VLA, use global #define
                #else
715
                               , LIMBS
                #endif
                           );
                        if (mu = -1.0 / 0.0)
                        {
720
                              output[k + 0] = 0;
                              output[k + 1] = 0;
                              output[k + 2] = 0;
                       }
                        else
725
                        {
                              output \, [\, k \, + \, 0\,] \, = \, 255 \, * \, (\, 0.5 \, - \, 0.5 \, * \, \cos \, (mu \, / \, 1 \, ) \,) \, ;
                              output\,[\,k\,+\,1\,]\,\,=\,\,255\ *\ (\,0.5\ -\,\,0.5\ *\ cos\,(mu\,\,/\,\,10\,\,\,)\,)\,;
                              output [k + 2] = 255 * (0.5 - 0.5 * cos(mu / 100));
730
                #ifndef __OPENCL_VERSION__
                 void fixed_dump
735
                 (FILE *f
                       const fixed_word_t *a, int ai, int af
                        for (int aw = -(ai - 1); aw <= 0; ++aw)
740
```

fixed-point .gitignore

```
fprintf(f, "%08x", fixed_read(a, ai, af, aw));
           if (aw < 0) fputc(', ', f);
        fputc('.', f);
745
         for (int aw = 1; aw \le af; ++aw)
           fprintf(f, "%08x", fixed_read(a, ai, af, aw));
           if (aw < af) fputc(', ', f);
750
      }
      void fixed_from_mpfr
      ( fixed_word_t *a, int ai, int af
755
        const mpfr_t x
        fixed_word_t b[ai + af];
        mpfr_t y;
760
         mpfr_init2(y, mpfr_get_prec(x));
         mpfr\_set(y, x, MPFR\_RNDN);
         fixed_zero(a, ai, af);
        double limb;
        do
765
         {
           limb = mpfr_get_d(y, MPFR_RNDN);
           mpfr_sub_d(y, y, limb, MPFR_RNDN);
           fixed_from_double(b, ai, af, limb);
           {\tt fixed\_add}\,({\tt a}\,,\ {\tt ai}\,,\ {\tt af}\,,\ {\tt a}\,,\ {\tt ai}\,,\ {\tt af}\,,\ {\tt b}\,,\ {\tt ai}\,,\ {\tt af}\,)\,;
        \} while (limb != 0);
770
        mpfr_clear(y);
      }
      void fixed_from_string
775
      ( fixed_word_t *a, int ai, int af
        const char *x
        mpfr_t y;
        mpfr_init2(y, (ai + af + 1) * sizeof(fixed_word_t) * CHAR_BIT);
780
         \label{eq:mpfr_set_str} mpfr\_set\_str\left(y\,,\ x\,,\ 10\,,\ MPFR\_RNDN\right);
         fixed\_from\_mpfr\left(a\,,\ ai\,,\ af\,,\ y\right);
         mpfr_clear(y);
785
      #endif
```

3 .gitignore

```
benchmark-be
benchmark-le
fixed-point.c.h
*.ppm
*.png
```

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4 LICENSE.md

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Version 3, 19 November 2007

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 275 provided you inform other peers where the object code and
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5 Makefile

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all: benchmark-be benchmark-le

```
benchmark-be: benchmark.cc fixed-point.c fixed-point.c.h g++ -std=c++11 -pedantic -Wall -Wextra -Wno-vla -O3 -march=native -\swarrow fopenmp -o $@ benchmark.cc -lOpenCL -lmpfr -lm -DBIG_ENDIAN=1
```

benchmark-le: benchmark.cc fixed-point.c fixed-point.c.h g++ -std=c++11 -pedantic -Wall -Wextra -Wno-vla -O3 -march=native $-\ensuremath{\cancel{\cancel{L}}}$ fopenmp -o \$@ benchmark.cc -lOpenCL -lmpfr -lm -UBIG_ENDIAN

```
%.c.h: %.c xxd -i < $< $
```

6 README.md

```
# fixed-point
```

A simple fixed-point implementation in C99 / C11 / C++11+VLA / OpenCL, with a Mandelbrot set example.

benchmark

fixed-point README.md

```
This location can go to 1e-90 or so, need to increase iterations to
   200000 for that.
10
      make
      export re≥
         export im∠
         export zoom=1e-15
15
      export iterations=1000
      for host_endian in le be ; do
      for use_fixed in 0 1; do
      for use_cl in 0 1; do
      for device_bigendian in $(seq 0 $use_cl); do
        time ./benchmark-$host_endian 1920 1080 "$re" "$im" "$zoom" "$iterations" \ensuremath{\mathcal{L}}
20

    use_fixed}-${use_cl}-${device_bigendian}.ppm"

      done; done; done; done
   ## legal
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   released under GNU Affero General Public License
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   ## references
30
   - OpenCL C++ driver code example simplified from 'mandelbrot_cl.cpp' from
   <http://distrustsimplicity.net/articles/mandelbrot-speed-comparison/>
```