lyapunov-fm

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Contents

1 COPYING

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35 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Changes made by SMS for thesis style
%   no emphasis on "et al."
%   "Ph.D." includes periods (not "PhD")
%   moved year to immediately after author's name
40 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
ENTRY
  { address
```

```
      author
      booktitle
      chapter
      edition
      editor
      howpublished
      institution
      journal
      key
      month
      note
      number
      organization
      pages
      publisher
      school
      series
      title
      type
      volume
      year
    }
    {}
    { label extra.label sort.label }

INTEGERS { output.state before.all mid.sentence after.sentence after.block }

FUNCTION {init.state.consts}
{ #0 'before.all :=
  #1 'mid.sentence :=
  #2 'after.sentence :=
  #3 'after.block :=
}

STRINGS { s t }

FUNCTION {output.nonnull}
{ 's :=
  output.state mid.sentence =
    { ", " * write$ }
    { output.state after.block =
        { add.period$ write$
          newline$
          "\newblock " write$
        }
        { output.state before.all =
            'write$
            { add.period$ " " * write$ }
          if$
        }
      if$
      mid.sentence 'output.state :=
    }
  if$
  s
}
```

```
FUNCTION {output}
{ duplicate$ empty$
    'pop$
    'output.nonnull
  if$
}

FUNCTION {output.check}
{ 't :=
  duplicate$ empty$
    { pop$ "empty " t * " in " * cite$ * warning$ }
    'output.nonnull
  if$
}

FUNCTION {output.bibitem}
{ newline$

  "\bibitem[" write$
  label write$
  "]{" write$

  cite$ write$
  "}" write$
  newline$
  ""
  before.all 'output.state :=
}

FUNCTION {fin.entry}
{ add.period$
  write$
  newline$
}

FUNCTION {new.block}
{ output.state before.all =
    'skip$
    { after.block 'output.state := }
  if$
}

FUNCTION {new.sentence}
{ output.state after.block =
    'skip$
    { output.state before.all =
        'skip$
        { after.sentence 'output.state := }
      if$
    }
  if$
}

FUNCTION {not}
{   { #0 }
    { #1 }
  if$
```

```
}

FUNCTION {and}
{   'skip$
    { pop$ #0 }
  if$
}

FUNCTION {or}
{   { pop$ #1 }
    'skip$
  if$
}

FUNCTION {new.block.checka}
{ empty$
    'skip$
    'new.block
  if$
}

FUNCTION {new.block.checkb}
{ empty$
  swap$ empty$
  and
    'skip$
    'new.block
  if$
}

FUNCTION {new.sentence.checka}
{ empty$
    'skip$
    'new.sentence
  if$
}

FUNCTION {new.sentence.checkb}
{ empty$
  swap$ empty$
  and
    'skip$
    'new.sentence
  if$
}

FUNCTION {field.or.null}
{ duplicate$ empty$
    { pop$ "" }
    'skip$
  if$
}

FUNCTION {emphasize}
{ duplicate$ empty$
    { pop$ "" }
    { "{\em " swap$ * "}" * }
```

```
    if$
}

INTEGERS { nameptr namesleft numnames }

FUNCTION {format.names}
{ 's :=
  #1 'nameptr :=
  s num.names$ 'numnames :=
  numnames 'namesleft :=
    { namesleft #0 > }

    { s nameptr "{ff~}{vv~}{ll}{, jj}" format.name$ 't :=

      nameptr #1 >
        { namesleft #1 >
            { ", " * t * }
            { numnames #2 >
                { "," * }
                'skip$
              if$
              t "others" =
                { " et~al." * }
                { " and " * t * }
              if$
            }
          if$
        }
        't
      if$
      nameptr #1 + 'nameptr :=
      namesleft #1 - 'namesleft :=
    }
  while$
}

FUNCTION {format.authors}
{ author empty$
    { "" }
    { author format.names }
  if$
}

FUNCTION {format.editors}
{ editor empty$
    { "" }
    { editor format.names
      editor num.names$ #1 >
        { ", editors" * }
        { ", editor" * }
      if$
    }
  if$
}

FUNCTION {format.title}
{ title empty$
```

```
      { "" }

      { title "t" change.case$ }

275   if$
    }

    FUNCTION {n.dashify}
    { 't :=
280   ""
      { t empty$ not }
      { t #1 #1 substring$ "-" =
        { t #1 #2 substring$ "--" = not
          { "--" *
285         t #2 global.max$ substring$ 't :=
          }
          {   { t #1 #1 substring$ "-" = }
              { "-" *
                t #2 global.max$ substring$ 't :=
290           }
            while$
          }
        if$
        }
295     { t #1 #1 substring$ *
        t #2 global.max$ substring$ 't :=
        }
      if$
      }
300   while$
    }

    FUNCTION {format.date}
    { year empty$
305   { month empty$
        { "" }
        { "there's a month but no year in " cite$ * warning$
          month
        }
310     if$
      }
      { month empty$
        { "" }
        { month }
315     if$
      }
    if$
    }

320 FUNCTION {format.btitle}
    { title emphasize
    }

    FUNCTION {tie.or.space.connect}
325 { duplicate$ text.length$ #3 <
      { "~" }
      { " " }
```

```
    if$
    swap$ * *
}

FUNCTION {either.or.check}
{ empty$
    'pop$
    { "can't use both " swap$ * " fields in " * cite$ * warning$ }
  if$
}

FUNCTION {format.bvolume}
{ volume empty$
    { "" }
    { "volume" volume tie.or.space.connect
      series empty$
        'skip$
        { " of " * series emphasize * }
      if$
      "volume and number" number either.or.check
    }
  if$
}

FUNCTION {format.number.series}
{ volume empty$
    { number empty$
        { series field.or.null }
        { output.state mid.sentence =
            { "number" }
            { "Number" }
          if$
          number tie.or.space.connect
          series empty$
            { "there's a number but no series in " cite$ * warning$ }
            { " in " * series * }
          if$
        }
      if$
    }
    { "" }
  if$
}

FUNCTION {format.edition}
{ edition empty$
    { "" }
    { output.state mid.sentence =
        { edition "l" change.case$ " edition" * }
        { edition "t" change.case$ " edition" * }
      if$
    }
  if$
}

INTEGERS { multiresult }
```

```
FUNCTION {multi.page.check}
{ 't :=
  #0 'multiresult :=
    { multiresult not
      t empty$ not
      and
    }
    { t #1 #1 substring$
      duplicate$ "-" =
      swap$ duplicate$ "," =
      swap$ "+" =
      or or
        { #1 'multiresult := }
        { t #2 global.max$ substring$ 't := }
      if$
    }
  while$
  multiresult
}

FUNCTION {format.pages}
{ pages empty$
    { "" }
    { pages multi.page.check
        { "pages" pages n.dashify tie.or.space.connect }
        { "page" pages tie.or.space.connect }
      if$
    }
  if$
}

FUNCTION {format.year.label}
{ year extra.label *
}

FUNCTION {format.vol.num.pages}
{ volume field.or.null
  number empty$
    'skip$
    { "(" number * ")" * *
      volume empty$
        { "there's a number but no volume in " cite$ * warning$ }
        'skip$
      if$
    }
  if$
  pages empty$
    'skip$
    { duplicate$ empty$
        { pop$ format.pages }
        { ":" * pages n.dashify * }
      if$
    }
  if$
}

FUNCTION {format.chapter.pages}
```

```
{ chapter empty$
    'format.pages
    { type empty$
        { "chapter" }
        { type "l" change.case$ }
      if$
      chapter tie.or.space.connect
      pages empty$
        'skip$
        { ", " * format.pages * }
      if$
    }
  if$
}

FUNCTION {format.in.ed.booktitle}
{ booktitle empty$
    { "" }
    { editor empty$
        { "In " booktitle emphasize * }
        { "In " format.editors * ", " * booktitle emphasize * }
      if$
    }
  if$
}

FUNCTION {empty.misc.check}
{ author empty$ title empty$ howpublished empty$
  month empty$ year empty$ note empty$
  and and and and and

  key empty$ not and

    { "all relevant fields are empty in " cite$ * warning$ }
    'skip$
  if$
}

FUNCTION {format.thesis.type}
{ type empty$
    'skip$
    { pop$
      type "t" change.case$
    }
  if$
}

FUNCTION {format.tr.number}
{ type empty$
    { "Technical Report" }
    'type
  if$
  number empty$
    { "t" change.case$ }
    { number tie.or.space.connect }
  if$
}
```

```
FUNCTION {format.article.crossref}
{ key empty$
    { journal empty$
        { "need key or journal for " cite$ * " to crossref " * crossref *
          warning$
          ""
        }
        { "In {\em " journal * "\/}" * }
      if$
    }
    { "In " key * }
  if$
  " \cite{" * crossref * "}" *
}

FUNCTION {format.crossref.editor}
{ editor #1 "{vv~}{ll}" format.name$
  editor num.names$ duplicate$
  #2 >
    { pop$ " et~al." * }
    { #2 <
        'skip$
        { editor #2 "{ff }{vv }{ll}{ jj}" format.name$ "others" =
            { " et~al." * }
            { " and " * editor #2 "{vv~}{ll}" format.name$ * }
          if$
        }
      if$
    }
  if$
}

FUNCTION {format.book.crossref}
{ volume empty$
    { "empty volume in " cite$ * "'s crossref of " * crossref * warning$
      "In "
    }
    { "Volume" volume tie.or.space.connect
      " of " *
    }
  if$
  editor empty$
  editor field.or.null author field.or.null =
  or
    { key empty$
        { series empty$
            { "need editor, key, or series for " cite$ * " to crossref " *
              crossref * warning$
              "" *
            }
            { "{\em " * series * "\/}" * }
          if$
        }
        { key * }
      if$
    }
```

```
      { format.crossref.editor * }
    if$
    " \cite{" * crossref * "}" *
  }

  FUNCTION {format.incoll.inproc.crossref}
  { editor empty$
    editor field.or.null author field.or.null =
    or
      { key empty$
          { booktitle empty$
              { "need editor, key, or booktitle for " cite$ * " to crossref " *
                crossref * warning$
                ""
              }
              { "In {\em " booktitle * "\/}" * }
            if$
          }
          { "In " key * }
        if$
      }
      { "In " format.crossref.editor * }
    if$
    " \cite{" * crossref * "}" *
  }

  FUNCTION {article}
  { output.bibitem
    format.authors "author" output.check
    new.block
    format.year.label "year" output.check
    new.block
    format.title "title" output.check
    new.block
    crossref missing$
      { journal emphasize "journal" output.check
        format.vol.num.pages output
        format.date output
      }
      { format.article.crossref output.nonnull
        format.pages output
      }
    if$
    new.block
    note output
    fin.entry
  }

  FUNCTION {book}
  { output.bibitem
    author empty$
      { format.editors "author and editor" output.check }
      { format.authors output.nonnull
        crossref missing$
          { "author and editor" editor either.or.check }
          'skip$
        if$
```

```
      }
    if$
    new.block
    format.year.label "year" output.check
    new.block
    format.btitle "title" output.check
    crossref missing$
      { format.bvolume output
        new.block
        format.number.series output
        new.sentence
        publisher "publisher" output.check
        address output
      }
      { new.block
        format.book.crossref output.nonnull
      }
    if$
    format.edition output
    format.date output
    new.block
    note output
    fin.entry
}

FUNCTION {booklet}
{ output.bibitem
    format.authors output
    new.block
    format.year.label "year" output.check
    new.block
    format.title "title" output.check
    howpublished address new.block.checkb
    howpublished output
    address output
    format.date output
    new.block
    note output
    fin.entry
}

FUNCTION {inbook}
{ output.bibitem
    author empty$
      { format.editors "author and editor" output.check }
      { format.authors output.nonnull
        crossref missing$
          { "author and editor" editor either.or.check }
          'skip$
        if$
      }
    if$
    format.year.label "year" output.check
    new.block
    new.block
    format.btitle "title" output.check
    crossref missing$
```

```
      { format.bvolume output
        format.chapter.pages "chapter and pages" output.check
        new.block
        format.number.series output
        new.sentence
        publisher "publisher" output.check
        address output
      }
      { format.chapter.pages "chapter and pages" output.check
        new.block
        format.book.crossref output.nonnull
      }
    if$
    format.edition output
    format.date output
    new.block
    note output
    fin.entry
  }

  FUNCTION {incollection}
  { output.bibitem
    format.authors "author" output.check
    new.block
    format.year.label "year" output.check
    new.block
    format.title "title" output.check
    new.block
    crossref missing$
      { format.in.ed.booktitle "booktitle" output.check
        format.bvolume output
        format.number.series output
        format.chapter.pages output
        new.sentence
        publisher "publisher" output.check
        address output
        format.edition output
        format.date output
      }
      { format.incoll.inproc.crossref output.nonnull
        format.chapter.pages output
      }
    if$
    new.block
    note output
    fin.entry
  }

  FUNCTION {inproceedings}
  { output.bibitem
    format.authors "author" output.check
    new.block
    format.year.label "year" output.check
    new.block
    format.title "title" output.check
    new.block
    crossref missing$
```

```
      { format.in.ed.booktitle "booktitle" output.check
        format.bvolume output
        format.number.series output
        format.pages output
        address empty$
          { organization publisher new.sentence.checkb
            organization output
            publisher output
            format.date output
          }
          { address output.nonnull
            format.date output
            new.sentence
            organization output
            publisher output
          }
        if$
      }
      { format.incoll.inproc.crossref output.nonnull
        format.pages output
      }
    if$
    new.block
    note output
    fin.entry
}

FUNCTION {conference} { inproceedings }

FUNCTION {manual}
{ output.bibitem
  author empty$
    { organization empty$
        'skip$
        { organization output.nonnull
          address output
        }
      if$
    }
    { format.authors output.nonnull }
  if$
  format.year.label "year" output.check
  new.block
  new.block
  format.btitle "title" output.check
  author empty$
    { organization empty$
        { address new.block.checka
          address output
        }
        'skip$
      if$
    }
    { organization address new.block.checkb
      organization output
      address output
    }
```

```
    if$
    format.edition output
    format.date output
    new.block
    note output
    fin.entry
}

FUNCTION {mastersthesis}
{ output.bibitem
    format.authors "author" output.check
    new.block
    format.year.label "year" output.check
    new.block
    format.title "title" output.check
    new.block
    "Master's thesis" format.thesis.type output.nonnull
    school "school" output.check
    address output
    format.date output
    new.block
    note output
    fin.entry
}

FUNCTION {misc}
{ output.bibitem
    format.authors output
    new.block
    format.year.label output
    new.block
    title howpublished new.block.checkb
    format.title output
    howpublished new.block.checka
    howpublished output
    format.date output
    new.block
    note output
    fin.entry
    empty.misc.check
}

FUNCTION {phdthesis}
{ output.bibitem
    format.authors "author" output.check
    new.block
    format.year.label "year" output.check
    new.block
    format.btitle "title" output.check
    new.block
    "{Ph.D.} thesis" format.thesis.type output.nonnull
    school "school" output.check
    address output
    format.date output
    new.block
    note output
    fin.entry
```

```
}

FUNCTION {proceedings}
{ output.bibitem
  editor empty$
    { organization output }
    { format.editors output.nonnull }
  if$
  new.block
  format.year.label "year" output.check
  new.block
  format.btitle "title" output.check
  format.bvolume output
  format.number.series output
  address empty$
    { editor empty$
        { publisher new.sentence.checka }
        { organization publisher new.sentence.checkb
          organization output
        }
      if$
      publisher output
      format.date output
    }
    { address output.nonnull
      format.date output
      new.sentence
      editor empty$
        'skip$
        { organization output }
      if$
      publisher output
    }
  if$
  new.block
  note output
  fin.entry
}

FUNCTION {techreport}
{ output.bibitem
  format.authors "author" output.check
  new.block
  format.year.label "year" output.check
  new.block
  format.title "title" output.check
  new.block
  format.tr.number output.nonnull
  institution "institution" output.check
  address output
  format.date output
  new.block
  note output
  fin.entry
}

FUNCTION {unpublished}
```

```
{ output.bibitem
  format.authors "author" output.check
  new.block
  format.year.label "year" output.check
  new.block
  format.title "title" output.check
  new.block
  note "note" output.check
  format.date output
  fin.entry
}

FUNCTION {default.type} { misc }

MACRO {jan} {"January"}

MACRO {feb} {"February"}

MACRO {mar} {"March"}

MACRO {apr} {"April"}

MACRO {may} {"May"}

MACRO {jun} {"June"}

MACRO {jul} {"July"}

MACRO {aug} {"August"}

MACRO {sep} {"September"}

MACRO {oct} {"October"}

MACRO {nov} {"November"}

MACRO {dec} {"December"}

MACRO {acmcs} {"ACM Computing Surveys"}

MACRO {acta} {"Acta Informatica"}

MACRO {cacm} {"Communications of the ACM"}

MACRO {ibmjrd} {"IBM Journal of Research and Development"}

MACRO {ibmsj} {"IBM Systems Journal"}

MACRO {ieeese} {"IEEE Transactions on Software Engineering"}

MACRO {ieeetc} {"IEEE Transactions on Computers"}

MACRO {ieeetcad}
 {"IEEE Transactions on Computer-Aided Design of Integrated Circuits"}

MACRO {ipl} {"Information Processing Letters"}
```

```
MACRO {jacm} {"Journal of the ACM"}

MACRO {jcss} {"Journal of Computer and System Sciences"}

MACRO {scp} {"Science of Computer Programming"}

MACRO {sicomp} {"SIAM Journal on Computing"}

MACRO {tocs} {"ACM Transactions on Computer Systems"}

MACRO {tods} {"ACM Transactions on Database Systems"}

MACRO {tog} {"ACM Transactions on Graphics"}

MACRO {toms} {"ACM Transactions on Mathematical Software"}

MACRO {toois} {"ACM Transactions on Office Information Systems"}

MACRO {toplas} {"ACM Transactions on Programming Languages and Systems"}

MACRO {tcs} {"Theoretical Computer Science"}

READ

FUNCTION {sortify}
{ purify$
  "l" change.case$
}

INTEGERS { len }

FUNCTION {chop.word}
{ 's :=
  'len :=
  s #1 len substring$ =
    { s len #1 + global.max$ substring$ }
    's
  if$
}

INTEGERS { et.al.char.used }

FUNCTION {initialize.et.al.char.used}
{ #0 'et.al.char.used :=
}

EXECUTE {initialize.et.al.char.used}

FUNCTION {format.lab.names}
{ 's :=
  s num.names$ 'numnames :=

  numnames #1 =
    { s #1 "{vv }{ll}" format.name$ }
    { numnames #2 =
        { s #1 "{vv }{ll }and " format.name$ s #2 "{vv }{ll}" format.name$ *
        }
```

```
          { s #1 "{vv }{ll }\bgroup et al.\egroup " format.name$ }
        if$
      }
    if$

}

FUNCTION {author.key.label}
{ author empty$
    { key empty$

        { cite$ #1 #3 substring$ }

        { key #3 text.prefix$ }
      if$
    }
    { author format.lab.names }
  if$
}

FUNCTION {author.editor.key.label}
{ author empty$
    { editor empty$
        { key empty$

            { cite$ #1 #3 substring$ }

            { key #3 text.prefix$ }
          if$
        }
        { editor format.lab.names }
      if$
    }
    { author format.lab.names }
  if$
}

FUNCTION {author.key.organization.label}
{ author empty$
    { key empty$
        { organization empty$

            { cite$ #1 #3 substring$ }

            { "The " #4 organization chop.word #3 text.prefix$ }
          if$
        }
        { key #3 text.prefix$ }
      if$
    }
    { author format.lab.names }
  if$
}

FUNCTION {editor.key.organization.label}
{ editor empty$
    { key empty$
```

```
        { organization empty$
1070
            { cite$ #1 #3 substring$ }

            { "The " #4 organization chop.word #3 text.prefix$ }
          if$
1075      }
          { key #3 text.prefix$ }
        if$
      }
      { editor format.lab.names }
1080  if$
}

FUNCTION {calc.label}
{ type$ "book" =
1085  type$ "inbook" =
  or
    'author.editor.key.label
    { type$ "proceedings" =
        'editor.key.organization.label
1090    { type$ "manual" =
            'author.key.organization.label
            'author.key.label
          if$
        }
1095    if$
    }
  if$
  duplicate$

1100  "\protect\citename{" swap$ * "}" *
  year field.or.null purify$ *
  'label :=
  year field.or.null purify$ *

1105  sortify 'sort.label :=
}

FUNCTION {sort.format.names}
{ 's :=
1110  #1 'nameptr :=
  ""
  s num.names$ 'numnames :=
  numnames 'namesleft :=
    { namesleft #0 > }
1115    { nameptr #1 >
        { "   " * }
        'skip$
      if$

1120    s nameptr "{vv{ } }{ll{ }}{  ff{ }}{  jj{ }}" format.name$ 't :=

      nameptr numnames = t "others" = and
        { "et al" * }
        { t sortify * }
1125    if$
```

```
      nameptr #1 + 'nameptr :=
      namesleft #1 - 'namesleft :=
    }
  while$
}

FUNCTION {sort.format.title}
{ 't :=
  "A " #2
    "An " #3
      "The " #4 t chop.word
    chop.word
  chop.word
  sortify
  #1 global.max$ substring$
}

FUNCTION {author.sort}
{ author empty$
    { key empty$
        { "to sort, need author or key in " cite$ * warning$
          ""
        }
        { key sortify }
      if$
    }
    { author sort.format.names }
  if$
}

FUNCTION {author.editor.sort}
{ author empty$
    { editor empty$
        { key empty$
            { "to sort, need author, editor, or key in " cite$ * warning$
              ""
            }
            { key sortify }
          if$
        }
        { editor sort.format.names }
      if$
    }
    { author sort.format.names }
  if$
}

FUNCTION {author.organization.sort}
{ author empty$
    { organization empty$
        { key empty$
            { "to sort, need author, organization, or key in " cite$ * warning$
              ""
            }
            { key sortify }
          if$
        }
```

```
          { "The " #4 organization chop.word sortify }
        if$
      }
      { author sort.format.names }
    if$
}

FUNCTION {editor.organization.sort}
{ editor empty$
    { organization empty$
        { key empty$
            { "to sort, need editor, organization, or key in " cite$ * warning$
              ""
            }
            { key sortify }
          if$
        }
        { "The " #4 organization chop.word sortify }
      if$
    }
    { editor sort.format.names }
  if$
}

FUNCTION {presort}

{ calc.label
  sort.label
  "    "
  *
  type$ "book" =

  type$ "inbook" =
  or
    'author.editor.sort
    { type$ "proceedings" =
        'editor.organization.sort
        { type$ "manual" =
            'author.organization.sort
            'author.sort
          if$
        }
      if$
    }
  if$

  *

  "    "
  *
  year field.or.null sortify
  *
  "    "
  *
  title field.or.null
  sort.format.title
  *
```

```
  #1 entry.max$ substring$
  'sort.key$ :=
}

ITERATE {presort}

SORT

STRINGS { longest.label last.sort.label next.extra }

INTEGERS { longest.label.width last.extra.num }

FUNCTION {initialize.longest.label}
{ "" 'longest.label :=
  #0 int.to.chr$ 'last.sort.label :=
  "" 'next.extra :=
  #0 'longest.label.width :=
  #0 'last.extra.num :=
}

FUNCTION {forward.pass}
{ last.sort.label sort.label =
    { last.extra.num #1 + 'last.extra.num :=
      last.extra.num int.to.chr$ 'extra.label :=
    }
    { "a" chr.to.int$ 'last.extra.num :=
      "" 'extra.label :=
      sort.label 'last.sort.label :=
    }
  if$
}

FUNCTION {reverse.pass}
{ next.extra "b" =
    { "a" 'extra.label := }
    'skip$
  if$
  label extra.label * 'label :=
  label width$ longest.label.width >
    { label 'longest.label :=
      label width$ 'longest.label.width :=
    }
    'skip$
  if$
  extra.label 'next.extra :=
}

EXECUTE {initialize.longest.label}

ITERATE {forward.pass}

REVERSE {reverse.pass}

FUNCTION {begin.bib}

{ et.al.char.used
    { "\newcommand{\etalchar}[1]{$^{#1}$}" write$ newline$ }
```

```
      'skip$
    if$
    preamble$ empty$
1300
      'skip$
      { preamble$ write$ newline$ }
    if$

1305  "\begin{thebibliography}{" "}" * write$ newline$

}

EXECUTE {begin.bib}
1310
EXECUTE {init.state.consts}

ITERATE {call.type$}

1315 FUNCTION {end.bib}
{ newline$
  "\end{thebibliography}" write$ newline$
}

1320 EXECUTE {end.bib}
```

3 doc/figure1.pd

```
#N canvas 0 0 169 102 36;
#N canvas 0 0 1905 1014 chaos 1;
#X obj 164 356 *~;
#X obj 164 410 +~;
 5 #X obj 164 464 mtof~;
#X obj 164 518 osc~;
#X obj 381 356 *~;
#X obj 381 410 +~;
#X obj 381 464 mtof~;
10 #X obj 381 518 osc~;
#X obj 616 100 block~ 2;
#X obj 164 100 table x 2;
#X obj 381 100 table y 2;
#N canvas 0 576 212 78 receive 0;
15 #X obj 72 184 tabreceive~ y;
#X obj 444 184 tabreceive~ x;
#X obj 72 268 outlet~;
#X obj 444 268 outlet~;
#X connect 0 0 2 0;
20 #X connect 1 0 3 0;
#X restore 164 282 pd receive;
#N canvas 0 754 212 76 send 0;
#X obj 80 108 inlet~;
#X obj 80 192 tabsend~ x;
25 #X obj 456 108 inlet~;
#X obj 456 192 tabsend~ y;
#X connect 0 0 1 0;
#X connect 2 0 3 0;
#X restore 164 592 pd send;
30 #X obj 244 410 r f_x;
```

```
#X obj 244 356 r m_x;
#X obj 461 356 r m_y;
#X obj 461 410 r f_y;
#X obj 204 174 tabreceive~ y;
#X obj 506 174 tabreceive~ x;
#X obj 204 228 outlet~;
#X obj 506 228 outlet~;
#X obj 204 646 inlet~;
#X obj 204 700 tabsend~ x;
#X obj 506 646 inlet~;
#X obj 506 700 tabsend~ y;
#X connect 0 0 1 0;
#X connect 1 0 2 0;
#X connect 2 0 3 0;
#X connect 3 0 12 0;
#X connect 4 0 5 0;
#X connect 5 0 6 0;
#X connect 6 0 7 0;
#X connect 7 0 12 1;
#X connect 11 0 0 0;
#X connect 11 1 4 0;
#X connect 13 0 1 1;
#X connect 14 0 0 1;
#X connect 15 0 4 1;
#X connect 16 0 5 1;
#X connect 17 0 19 0;
#X connect 18 0 20 0;
#X connect 21 0 22 0;
#X connect 23 0 24 0;
#X restore 172 168 pd chaos;
#X floatatom 80 92 4 0 0 2 f_x - -;
#X floatatom 200 92 4 0 0 2 f_y - -;
#X floatatom 320 92 4 0 0 2 m_x - -;
#X floatatom 440 92 4 0 0 2 m_y - -;
#X obj 212 248 dac~;
```

4 doc/lac2013.sty

```
%
% lac2013.sty
%
\typeout{}
\typeout{LAC2013 Proceedings style -- Sep 2012}
%\typeout{switch back to A4-Letter format}
%\typeout{LAC2012 Proceedings style -- Jan 2012}
%\typeout{updated for US-Letter format}
%\typeout{LAC2012 Proceedings style -- October 2011}
%\typeout{only slightly different from}
%\typeout{LAC2011 Proceedings style -- October 2010}
%\typeout{only slightly different from}
%\typeout{LAC2010 Proceedings style -- November 2009}
%\typeout{only slightly different from}
%\typeout{LAC2009 Proceedings style -- October 2008}
%\typeout{only slightly different from}
%\typeout{LAC2008 Proceedings style -- September 2007}
%\typeout{only slightly different from}
%\typeout{LAC2007 Proceedings style -- September 2006}
```

```
%\typeout{only slightly different from}
%\typeout{LAC2005 Proceedings style -- November 2005}
%\typeout{only slightly different from}
%\typeout{Coling 2004 Proceedings style -- February 2004}
%\typeout{only slightly different from}
%\typeout{ACL-COLING 1998 Proceedings style -- March 31st 1998}
\typeout{}
%
% ---------------------------------------------------------------------
%
% This is the LaTeX style file for LAC2012 (lac2012.sty).
% It is an almost unmodified copy of colacl.sty, the style file
% of Coling (International Conference on Computational Linguistics).
% Many thanks to Coling for their permission to use it!
% The original may be found here:
% http://www.issco.unige.ch/coling2004/coling2004downloads.html
%
% ---------------------------------------------------------------------
%
% This is the LaTeX style file for ACL-COLING 1998.  It is based on
% a series of similar files prepared for previous conferences by
% Fernando Pereira, Paul Jacobs, Stuart Shieber, Peter F.
% Patel-Schneider and others.  Various changes have been made, chiefly
% to save space in the final output or remove redundant definitions.
%
% colacl.sty is designed for use as a package or option with the
% standard LaTeX article class, and the BibTeX style acl.bst.
%
% Author/title and citation formatting differs slightly from standard
% LaTeX; see AUTHOR FORMATS and CITATION FORMATS below for more
% information.
%
% This file is supplied as a hopefully convenient implementation of
% some of the "instructions for authors" repeated below.  It is not
% guaranteed to work in any given LaTeX installation or in conjunction
% with any given class, package or style, and it is not intended as
% a LaTeX tutorial.
%
% ---------------------------------------------------------------------
% Instructions for authors
%
%   (i) Maximum length of full papers: 8 pages.
%
%  (ii) Paper size: A4.
%
% (iii) Margins: set so that text lies within a rectangle 9in (23cm)
%       high and 6.5in (16.5cm) wide.
%
%  (iv) Body of text to be set in two columns.  Full-width figures
%       (i.e. using \begin{figure*}) and tables may be used if
%       necessary.
%
%   (v) Use standard fonts, e.g. Computer Modern Roman, Times Roman, no
%       smaller than 10pt.
%
%  (vi) No page numbering.
%
```

```
% Items (iii), (iv) and (vi) are handled by this file, and should
% therefore not be overridden by resetting \textwidth, \textheight,
% \pagestyle etc. in your document, or by using styles or packages
% which have the same effect.
%
% ---------------------------------------------------------------------
% To convert papers prepared with colaclsub.sty to the final format
% for use with colacl.sty:
%
% (1) Remove commands specific to the original submission format
%     (\type, \subject, \contact, \conference, \makeidpage).
%
% (2) Replace \summary{...} with an abstract, using the normal
%     abstract environment, placed after \maketitle.
%
% A simple document template:
%
%   \documentclass[11pt,letterpaper]{article}
%   \usepackage{lac2012}
%   \title{...}
%   \author{...}              % see below for possible formats
%   \begin{document}
%   \maketitle
%   \begin{abstract}
%   ...                       % contents of abstract
%   \end{abstract}
%   ...                       % contents of article
%   \bibliographystyle{acl}  % use acl.bst
%   \bibliography{...}
%   \end{document}
%
% Users of obsolete LaTeX versions can try:
%
%   \documentstyle[lac2012]{article}    % or [11pt,lac2012]
%   \title{...}
%   ...
%
%
% ---------------------------------------------------------------------
% AUTHOR FORMATS
%
% Author information can be set in various styles.
%
% For several authors from the same institution:
% \author{Author 1 \and ... \and Author n \\
%         Address line \\ ... \\ Address line}
% if the names do not fit well on one line use
%         Author 1 \\ {\bf Author 2} \\ ... \\ {\bf Author n} \\
%
% For authors from different institutions:
% \author{Author 1 \\ Address line \\  ... \\ Address line
%         \And  ... \And
%         Author n \\ Address line \\ ... \\ Address line}
%
% To start a separate "row" of authors use \AND, as in
% \author{Author 1 \\ Address line \\  ... \\ Address line
%         \AND
```

```
%          Author 2 \\ Address line \\ ... \\ Address line \And
%          Author 3 \\ Address line \\ ... \\ Address line}
%
% If the title and author information does not fit in the area allocated,
% place \setlength\titlebox{<new height>} after \usepackage{colacl},
% where <new height> can be something larger than 2.0in
%
% ---------------------------------------------------------------------
% CITATION FORMATS
%
% Three possible citation formats:
% "\cite{...}"      produces a citation like "(Author, 1999)"
% "\shortcite{...}" produces a citation like "(1999)"
% "\newcite{...}"   produces a citation like "Author (1999)"
%
% All three take an optional argument which can be used to add page
% references, etc.:
% "\newcite[1--6]{...}" produces a citation like "Author (1999, 1--6)"
%
% ---------------------------------------------------------------------
% IF IT DOESN'T WORK
%
% The error message "File `colacl.sty' not found." indicates that this
% file has not been installed in a location which is visible to your
% LaTeX.  Try putting it in the same directory as your paper, and
% running LaTeX there.  Consult your `Local Guide' documentation or
% your system administrator to find out how LaTeX searches for input
% files.
%
% "\documentclass..." is a LaTeX2e declaration.  An error message
% "Undefined control sequence." followed by a line ending in
% "\documentclass" indicates that you have used this with an obsolete
% LaTeX installation.  Use the "\documentstyle" variant shown above.
%
% As a last resort, forget about colacl.sty and simply copy the
% following lines (uncommented, obviously) into your document before
% the "\begin{document}":
%
% \setlength\topmargin{0.0in}
% \setlength\oddsidemargin{-0.0in}
% \setlength\textheight{9.0in}
% \setlength\textwidth{6.5in}
% \setlength\columnsep{0.25in}
% \setlength\headheight{0pt}
% \setlength\headsep{0pt}
% \thispagestyle{empty}
% \pagestyle{empty}
% \flushbottom
% \twocolumn
% \sloppy
%
% Some interactions with other packages may still occur.  In order to
% remove the page number from the first page, you may have to place the
% "\thispagestyle{empty}" command immediately after "\maketitle".
%
% ---------------------------------------------------------------------
% NOTE:  Some laser printers have a serious problem printing TeX output.
```

```
% These printing devices, commonly known as "write-white" laser
% printers, tend to make characters too light.  To get around this
% problem, a darker set of fonts must be created for these devices.
%
% ---------------------------------------------------------------------
% Physical page layout - slightly modified from IJCAI by pj
% Physical page layout - slightly modified by rg for LAC
\newlength\myavierh
\setlength\myavierh{297mm}
%\message{The page height is \the\paperheight}
\ifdim \paperheight = \myavierh
\message{ !! Using ISO-A4 paper}
%% A4paper
\setlength\topmargin{0.0in}
\setlength\oddsidemargin{-0.0in}
\setlength\textheight{9.5in}   %%sm was 9.0in
\setlength\textwidth{6.5in}
\setlength\columnsep{0.25in}
\newlength\titlebox
\setlength\titlebox{2.0in}              % was 2.25in
\setlength\headheight{0pt}
\setlength\headsep{0pt}
\setlength\footskip{0pt}                % irrelevant when no footers.

\else
%% US-Letter
\message{ !! Using US-Letter paper}
\setlength{\pdfpagewidth}{8.5in}
\setlength{\pdfpageheight}{11in}
\setlength\topmargin{0.0in}
\setlength\oddsidemargin{-0.2in}
\setlength\textheight{8.8in}
\setlength\textwidth{6.9in}
\setlength\columnsep{0.4in}
\newlength\titlebox
\setlength\titlebox{2.1in}
\setlength\headheight{0pt}
\setlength\headsep{0pt}
\setlength\footskip{0pt}      % irrelevant when no footers.
\fi

\pagestyle{empty}                       % no page numbers
\thispagestyle{empty}                   % no page numbers
\flushbottom
\twocolumn
\sloppy

% We're never going to need a table of contents, so just flush it to
% save space --- suggested by drstrip@sandia-2
\def\addcontentsline#1#2#3{}

% ---------------------------------------------------------------------
% Title stuff, taken from deproc.

\def\maketitle{%
  \par%
  \begingroup%
```

```
      \def\thefootnote{\fnsymbol{footnote}}%
      \def\@makefnmark{\rlap{$^{\@thefnmark}$\hss}}%
      % no paragraph breaks in \thanks
      \long\def\@makefntext##1{%
                  \parindent 1em\noindent%
                  \hbox to 1em{$^{\@thefnmark}$}##1}
      \twocolumn[\@maketitle] \@thanks%
  \endgroup%
  \setcounter{footnote}{0}%
  \let\maketitle\relax\let\@maketitle\relax%
  \gdef\@thanks{}\gdef\@author{}\gdef\@title{}%
  \let\thanks\relax}

% some vertical space removed here: skip above and below title
%
\def\@maketitle{%
  \vbox to \titlebox{%
    \hsize\textwidth\linewidth\hsize%
    \vskip 0.125in minus 0.05in%
    \centering{\Large\bf \@title \par}%
    \vskip 0.2in plus 0.1fil minus 0.1in
    {\def\and{\unskip\enspace{\rm and}\enspace}%
     \def\And{\end{tabular}\hss \egroup \hskip 1in plus 2fil
              \hbox to 0pt\bgroup\hss \begin{tabular}[t]{c}\bf}%
     \def\AND{\end{tabular}\hss\egroup \hfil\hfil\egroup
              \vskip 0.25in plus 1fil minus 0.125in
              \hbox to \linewidth\bgroup\large \hfil\hfil
              \hbox to 0pt\bgroup\hss \begin{tabular}[t]{c}\bf}
    \hbox to \linewidth \bgroup\large \hfil\hfil
    \hbox to 0pt\bgroup\hss \begin{tabular}[t]{c}\bf\@author
                            \end{tabular}\hss\egroup
    \hfil\hfil\egroup}
  \vskip 0.3in plus 2fil minus 0.1in
}}

% ------------------------------------------------------------------------
% abstract, changed for LAC2005

% \renewenvironment{abstract}{\section*{\centerline{Abstract}}}{}
\renewenvironment{abstract}{\section*{Abstract}}{}

% ------------------------------------------------------------------------
% keywords, added for LAC2005

\def\keywords{\section*{Keywords}}

% ------------------------------------------------------------------------
% bibliography and citations

% most of cite format is from aclsub.sty by SMS

% don't box citations, separate with ; and a space
% Replaced for multiple citations (pj)
% don't box citations and also add space, semicolon between multiple
% citations
%
\def\@citex[#1]#2{\if@filesw\immediate\write\@auxout{\string\citation{#2}}\fi
```

```
305     \def\@citea{}\@cite{\@for\@citeb:=#2\do
          {\@citea\def\@citea{; }\@ifundefined
            {b@\@citeb}{{\bf ?}\@warning
             {Citation `\@citeb' on page \thepage \space undefined}}%
       {\csname b@\@citeb\endcsname}}}{#1}}
310
    % Allow short (name-less) citations, when used in
    % conjunction with a bibliography style that creates labels like
    %       \citename{<names>, }<year>
    %
315 \let\@internalcite\cite
    \def\cite{\def\citename##1{##1, }\@internalcite}
    \def\longcite{\def\citename##1{##1, }\@internalcite}

    \def\shortcite{\def\citename##1{}\@internalcite}
320 \def\newcite{\def\citename##1{{##1}\ (}\@internalciteb}

    % Macros for \newcite, which leaves name in running text, and is
    % otherwise like \shortcite.
    %
325 \def\@citexb[#1]#2{\if@filesw\immediate\write\@auxout{\string\citation{#2}}\fi
      \def\@citea{}\@newcite{\@for\@citeb:=#2\do
        {\@citea\def\@citea{;\penalty\@m\ }\@ifundefined
           {b@\@citeb}{{\bf ?}\@warning
           {Citation `\@citeb' on page \thepage \space undefined}}%
330 % gjr: hbox causes too many bad linebreaks
    %\hbox{\csname b@\@citeb\endcsname}}}{#1}}
         {\csname b@\@citeb\endcsname}}}{#1}}

    \def\@internalciteb{%
335   \@ifnextchar [{\@tempswatrue\@citexb}{\@tempswafalse\@citexb[]}}

    \def\@newcite#1#2{{#1\if@tempswa, #2\fi)}}

    % gjr: no labels in this bibliography style
340 % \def\@biblabel#1{\def\citename##1{##1}[#1]\hfill}
    \def\@biblabel#1{}

    % use square brackets after all
    %%% More changes made by SMS (originals in latex.tex)
345 % Use parentheses instead of square brackets in the text.
    % \def\@cite#1#2{({#1\if@tempswa , #2\fi})}

    % Don't put a label in the bibliography at all.  Just use the unlabeled format
    % instead.
350 % gjr: removed \@mkboth -- no headers here.
    % gjr: reduced vertical space between entries (plus was .33em)
    %

    \def\thebibliography#1{\section*{References}\list
355  {[\arabic{enumi}]}{\settowidth\labelwidth{[#1]}\leftmargin\labelwidth
     \advance\leftmargin\labelsep
     \usecounter{enumi}}
     \def\newblock{\hskip .11em plus .11em minus .07em}
     \sloppy\clubpenalty4000\widowpenalty4000
360  \sfcode`\.=1000\relax}
    \let\endthebibliography=\endlist
```

```
% \def\thebibliography#1{%
%   \section*{References}
%   \list{}{\setlength{\labelwidth}{0pt}
%           \setlength{\leftmargin}{\parindent}
%           \setlength{\itemsep}{0.11ex plus 0.11ex}
%           \setlength{\parsep}{0ex}
%           \setlength{\itemindent}{-\parindent}}
%   \def\newblock{\hskip .11em plus .11em minus -.07em}
%   \sloppy\clubpenalty4000\widowpenalty4000
%   \sfcode`\.=1000\relax}
% \let\endthebibliography=\endlist

% Allow for a bibliography of sources of attested examples
\def\thesourcebibliography#1{%
  \section*{Sources of Attested Examples}
  \list{}{\setlength{\labelwidth}{0pt}
          \setlength{\leftmargin}{\parindent}
          \setlength{\itemsep}{0.11ex plus 0.11ex}
          \setlength{\parsep}{0ex}
          \setlength{\itemindent}{-\parindent}}
  \def\newblock{\hskip .11em plus .11em minus -.07em}
  \sloppy\clubpenalty4000\widowpenalty4000
  \sfcode`\.=1000\relax}
\let\endthesourcebibliography=\endlist

\def\@lbibitem[#1]#2{\item[]\if@filesw
      { \def\protect##1{\string ##1\space}\immediate
        \write\@auxout{\string\bibcite{#2}{#1}}\fi\ignorespaces}}

\def\@bibitem#1{\item\if@filesw \immediate\write\@auxout
       {\string\bibcite{#1}{\the\c@enumi}}\fi\ignorespaces}

% ---------------------------------------------------------------------
% Section headings with less space

\def\section{%
    \@startsection{section}{1}{\z@}%
                  {-2.0ex plus -0.5ex minus -0.3ex}%
                  {0.8ex plus 0.3ex minus 0.2ex}%
                  {\large\bf\raggedright}}
\def\subsection{%
    \@startsection{subsection}{2}{\z@}%
                  {-1.4ex plus -0.4ex minus -0.2ex}%
                  {0.6ex plus 0.2ex minus 0.1ex}%
                  {\normalsize\bf\raggedright}}
\def\subsubsection{%
    \@startsection{subsubsection}{3}{\z@}%
                  {-0.8ex plus -0.3ex minus -0.1ex}%
                  {0.4ex plus 0.1ex minus 0.1ex}%
                  {\normalsize\bf\raggedright}}
\def\paragraph{%
    \@startsection{paragraph}{4}{\z@}%
                  {-0.8ex plus -0.3ex minus -0.1ex}%
                  {-1em}%
                  {\normalsize\bf}}
\def\subparagraph{%
```

```
    \@startsection{subparagraph}{5}{\parindent}%
420                 {0.4ex plus 0.3ex minus 0.1ex}%
                    {-1em}%
                    {\normalsize\bf}}


425 % ------------------------------------------------------------------
    % Footnotes

    %\footnotesep 6.65pt %
    %\skip\footins 9pt plus 4pt minus 2pt
430 %\def\footnoterule{\kern-3pt \hrule width 5pc \kern 2.6pt }
    %\setcounter{footnote}{0}

    % ------------------------------------------------------------------
    % Lists and paragraphs
435
    \setlength\parindent{1em}

    \leftmargin 2em \leftmargini\leftmargin \leftmarginii 2em
    \leftmarginiii 1.5em \leftmarginiv 1.0em \leftmarginv .5em \leftmarginvi .5em
440 \labelwidth\leftmargini\advance\labelwidth-\labelsep \labelsep 5pt

    % ------------------------------------------------------------------
    % Floats (figures, tables etc.)
    %
445 % Allow a larger proportion of the column/page to be taken up with
    % floats than the standard classes.  Also discourage the creation of
    % columns/pages containing only floats.

    % Maximum fraction of the page that can be occupied by floats:
450 %
    \renewcommand\topfraction{.9}
    \renewcommand\bottomfraction{.5}
    \renewcommand\dbltopfraction{.9}        % 2-column floats

455 % Minimum fraction of page that can be occupied by text:
    %
    \renewcommand\textfraction{.1}

    % Maximum fraction of a page that can be occupied by floats before a
460 % separate float page is produced:
    %
    \renewcommand\floatpagefraction{0.9}
    \renewcommand\dblfloatpagefraction{.9}  % 2-column floats

465 % ------------------------------------------------------------------
    %
    % Since we're using two columns, lines are short and we can get away
    % with less vertical space between lines, within lists and around
    % various kinds of display.
470 %
    % Normally, these parameters are set in the size option to the class
    % file (standard definitions are in classes.dtx).  Here we want to
    % accommodate 10pt, 11pt and 12pt, so we wrap the definitions in
    % \ifcase.
475 %
```

```
% \normalsize
%
\ifcase\@ptsize%
    \renewcommand{\normalsize}{%                                    10pt
        \@setsize\normalsize{11.3pt}\xpt\@xpt%
        \abovedisplayskip 10\p@\@plus2\p@\@minus5\p@%
        \abovedisplayshortskip\z@\@plus3\p@%
        \belowdisplayshortskip 4\p@\@plus3\p@\@minus3\p@%
        \belowdisplayskip\abovedisplayskip%
        \let\@listi\@listI}%
\or%
    \renewcommand{\normalsize}{%                                    11pt
        \@setsize\normalsize{12.6pt}\xipt\@xipt%
        \abovedisplayskip11\p@\@plus2\p@\@minus4\p@%
        \abovedisplayshortskip\z@\@plus3\p@%
        \belowdisplayshortskip5\p@\@plus3\p@\@minus2\p@%
        \belowdisplayskip\abovedisplayskip%
        \let\@listi\@listI}%
\or%
    \renewcommand{\normalsize}{%                                    12pt
        \@setsize\normalsize{13pt}\xiipt\@xiipt%
        \abovedisplayskip 11\p@ \@plus3\p@ \@minus5\p@%
        \abovedisplayshortskip \z@ \@plus3\p@%
        \belowdisplayshortskip 5\p@ \@plus3\p@ \@minus2\p@%
        \belowdisplayskip\abovedisplayskip%
        \let\@listi\@listI}%
\fi

% \small
%
\ifcase\@ptsize%
    \renewcommand{\small}{%                                         10pt
        \@setsize\small{10.5pt}\ixpt\@ixpt%
        \abovedisplayskip 8\p@ \@plus3\p@ \@minus3\p@%
        \abovedisplayshortskip \z@ \@plus2\p@%
        \belowdisplayshortskip 3\p@ \@plus2\p@ \@minus2\p@%
        \belowdisplayskip\abovedisplayskip%
        \def\@listi{\leftmargin\leftmargini%
                    \topsep 3.5\p@ \@plus1.5\p@ \@minus1.5\p@%
                    \parsep 1.5\p@ \@plus\p@ \@minus\p@%
                    \itemsep \parsep}}%
\or%
    \renewcommand{\small}{%                                         11pt
        \@setsize\small{11.3pt}\xpt\@xpt%
        \abovedisplayskip 9\p@ \@plus2\p@ \@minus4\p@%
        \abovedisplayshortskip \z@ \@plus3\p@%
        \belowdisplayshortskip 5\p@ \@plus2.5\p@ \@minus2.5\p@%
        \belowdisplayskip\abovedisplayskip%
        \def\@listi{\leftmargin\leftmargini%
                    \topsep 5\p@ \@plus2\p@ \@minus2\p@%
                    \parsep 2\p@ \@plus2\p@ \@minus\p@%
                    \itemsep \parsep}}%
\or%
    \renewcommand{\small}{%                                         12pt
        \@setsize\small{12pt}\xipt\@xipt%
        \abovedisplayskip 9\p@ \@plus3\p@ \@minus4\p@%
```

```
        \abovedisplayshortskip \z@ \@plus3\p@%
        \belowdisplayshortskip 5\p@ \@plus2.5\p@ \@minus2\p@%
        \belowdisplayskip\abovedisplayskip%
        \def\@listi{\leftmargin\leftmargini%
                    \topsep 5.5\p@ \@plus2.5\p@ \@minus2.5\p@%
                    \parsep 4\p@ \@plus1.5\p@ \@minus\p@%
                    \itemsep \parsep}}%
  \fi


  % \footnotesize
  %
  \ifcase\@ptsize
      \renewcommand{\footnotesize}{%                                  10pt
          \@setsize\footnotesize{9.3pt}\viiipt\@viiipt%
          \abovedisplayskip 5\p@ \@plus2\p@ \@minus3\p@%
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          \belowdisplayshortskip 2.5\p@\@plus\p@\@minus2\p@%
          \belowdisplayskip\abovedisplayskip%
          \def\@listi{\leftmargin\leftmargini%
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                      \parsep 1.5\p@ \@plus\p@ \@minus\p@%
                      \itemsep \parsep}}%
  \or%
      \renewcommand{\footnotesize}{%                                  11pt
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          \belowdisplayskip\abovedisplayskip%
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                      \itemsep \parsep}}%
  \fi


  % \large
  %
  \ifcase\@ptsize%
      \renewcommand{\large}{\@setsize\large{13pt}\xiipt\@xiipt}%   10pt
  \or%
      \renewcommand{\large}{\@setsize\large{13pt}\xiipt\@xiipt}%   11pt
  \or%
      \renewcommand{\large}{\@setsize\large{16pt}\xivpt\@xivpt}%   12pt
  \fi
```

```
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    %
    \ifcase\@ptsize%
        \renewcommand{\Large}{\@setsize\Large{16pt}\xivpt\@xivpt}%  10pt
595  \or%
        \renewcommand{\Large}{\@setsize\Large{16pt}\xivpt\@xivpt}%  11pt
     \or%
        \renewcommand{\Large}{\@setsize\Large{16pt}\xivpt\@xivpt}%  12pt
     \fi
600
    % Leave \scriptsize, \tiny, \huge, \Huge unchanged?

    %
    % Float separations, single and double-column
605 %
    \ifcase\@ptsize%
        \setlength\floatsep{10\p@ \@plus 2\p@ \@minus 2\p@}%       10pt
        \setlength\textfloatsep{16\p@ \@plus 2\p@ \@minus 4\p@}%
        \setlength\intextsep{10\p@ \@plus 2\p@ \@minus 2\p@}%
610     \setlength\dblfloatsep{10\p@ \@plus 2\p@ \@minus 2\p@}%
        \setlength\dbltextfloatsep{16\p@ \@plus 2\p@ \@minus 4\p@}%
     \or%
        \setlength\floatsep{10\p@ \@plus 2\p@ \@minus 2\p@}%       11pt
        \setlength\textfloatsep{16\p@ \@plus 2\p@ \@minus 4\p@}%
615     \setlength\intextsep{10\p@ \@plus 2\p@ \@minus 2\p@}%
        \setlength\dblfloatsep{10\p@ \@plus 2\p@ \@minus 2\p@}%
        \setlength\dbltextfloatsep{16\p@ \@plus 2\p@ \@minus 4\p@}%
     \or%
        \setlength\floatsep{12\p@ \@plus 3\p@ \@minus 3\p@}%       12pt
620     \setlength\textfloatsep{18\p@ \@plus 2\p@ \@minus 4\p@}%
        \setlength\intextsep{12\p@ \@plus 3\p@ \@minus 3\p@}%
        \setlength\dblfloatsep{12\p@ \@plus 2\p@ \@minus 4\p@}%
        \setlength\dbltextfloatsep{18\p@ \@plus 2\p@ \@minus 4\p@}%
     \fi
625
    %
    % Top-level list in \normalsize text
    %
    \ifcase\@ptsize%
630     \def\@listi{\leftmargin\leftmargini%                        10pt
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                    \itemsep  2.5\p@ \@plus\p@ \@minus0.5\p@}%
     \or%
635     \def\@listi{\leftmargin\leftmargini%                        11pt
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                    \parsep   3\p@ \@plus1.5\p@ \@minus\p@%
                    \itemsep  3\p@ \@plus1.5\p@ \@minus\p@}%
     \or%
640     \def\@listi{\leftmargin\leftmargini%                        12pt
                    \topsep   9\p@ \@plus3\p@    \@minus4\p@%
                    \parsep   4\p@   \@plus2\p@ \@minus\p@%
                    \itemsep  4\p@   \@plus2\p@ \@minus\p@}%
     \fi
645 \let\@listI\@listi
```

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%
% Embedded lists unchanged.
%


% ---------------------------------------------------------------------
% End of colacl.sty
% ---------------------------------------------------------------------
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5 doc/lyapunov-fm.bib

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author={Slater, Dan},
year={1998},
title={Chaotic {S}ound {S}ynthesis},
journal={Computer Music Journal},
volume={22},
number={2},
pages={12-19}
}

@article{Dewdney91,
author={Dewdney, A. K.},
year={1991},
title={Mathematical {R}ecreations: {L}eaping into {L}yapunov {S}pace},
journal={Scientific American},
volume={265},
pages={178-180}
}

@book{Elert07,
author={Elert, Glenn},
year={2007},
title={The Chaos Hypertextbook},
publisher={\url{http://hypertextbook.com/chaos/}}
}

@book{Falconer03,
author={Falconer, Kenneth},
year={2003},
title={Fractal Geometry: Mathematical Foundations and Applications, Second
    ↪ Edition},
publisher={Wiley}
}

@book{SoftRockEP,
author={ClaudiusMaximus},
year={2005},
title={Soft~Rock~EP},
publisher={\url{http://archive.org/details/ ClaudiusMaximus\_-\_Soft\_Rock\_EP}}
}

@book{SoftRockDVD,
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year={2006},
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publisher={\url{http://archive.org/details/ ClaudiusMaximus\_-\_Soft\_Rock\_DVD
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    ↪ }}
}

@book{OpenGL,
author={Segal, Mark},
author={Akeley, Kurt},
year={2013},
title={The OpenGL Graphics System: A Specification},
publisher={\url{http://www.opengl.org/registry/doc/ glspec43.core.20130214.pdf}}
}

@book{GLSL,
author={Kessenich, John},
author={Baldwin, Dave},
author={Rost, Randi},
author={LunarG},
year={2013},
title={The OpenGL Shading Language},
publisher={\url{http://www.opengl.org/registry/doc/ GLSLangSpec.4.30.8.pdf}}
}

@book{GLUT,
author={Kilgard, Mark J.},
year={1996},
title={The OpenGL Utility Toolkit (GLUT) Programming Interface},
publisher={\url{http://www.opengl.org/documentation/ specs/glut/glut-3.spec.pdf
    ↪ }}
}

@book{JACK,
author={Davis, Paul},
year={2013},
title={The JACK Audio Connection Kit},
publisher={\url{http://jackaudio.org}}
}
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6 doc/lyapunov-fm.tex

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\documentclass[11pt,a4paper]{article}
\usepackage{lac2013}
\usepackage{amsmath}
\usepackage{graphicx}
\usepackage{float}
\usepackage{dblfloatfix}
\usepackage{subfigure}
%\usepackage{hyperref}
\newcommand{\url}[1]{\texttt{#1}}
\sloppy
\newenvironment{contentsmall}{\small}

\DeclareMathOperator{\inc}{I}
\DeclareMathOperator{\wrap}{\%}
\DeclareMathOperator{\SR}{SR}

\title{Lyapunov Space of Coupled FM Oscillators}
\author
{Claude Heiland-Allen
```

```
20  \\ claude@mathr.co.uk
    }

    \begin{document}
    \maketitle
25
    \begin{abstract}
    \begin{contentsmall}
    Consider two coupled oscillators, each modulating the other's
    frequency. This system is governed by four parameters: the base
30  frequency and modulation index for each oscillator. For some
    parameter values the system becomes unstable. The Lyapunov
    exponent is used to measure the instability. Images of the
    parameter space are generated, with the number crunching implemented on graphics
    hardware using OpenGL. The mouse position over the displayed
35  image is linked to realtime audio output, creating an audio-visual browser
    for the 4D parameter space.
    \end{contentsmall}
    \end{abstract}

40  \keywords{
    \begin{contentsmall}
    chaos, DSP, GPU
    \end{contentsmall}
    }
45
    \begin{figure}[H]
    \centering
    \includegraphics[width=\textwidth]{example.png}
    \caption{Example output.}
50  \label{example}
    \end{figure}

    \section{Introduction}
    Soft Rock EP~\cite{SoftRockEP} and Soft Rock DVD~\cite{SoftRockDVD}
55  explored the transitions between order and chaos in coupled FM
    oscillators. A more recent continuation of
    this project is to make a map of the parameter space of coupled FM oscillators ↙
        ↳ on a perceptually
    relevant level and use it in live performance, choosing parameters on the basis
    of desired sound character.
60
    A bifurcation diagram produced by an analogue Moog
    synthesizer~\cite{Slater98} and images of Lyapunov fractals~\cite{Dewdney91}
    were inspiration to apply the latter technique to the parameter space
    of coupled FM oscillators in the digital realm.
65
    \newpage

    \section{Formulation}

70  \subsection{Coupled FM Oscillators}
    \begin{figure}[H]
    \centering
    \includegraphics[width=0.4\textwidth]{figure1.png}
    \caption{Coupled FM oscillators in Pure-data.}
75  \label{fig:pd}
```

```
\end{figure}
Consider the two coupled oscillators in Figure~\ref{fig:pd}.
Pure-data's model of interconnected components each with their
own internal state maps poorly to GPU architecture.  Considering
the whole system as one and flattening the internal state into
a single phase space vector leads to the following formulation
as a mutual recurrence relation:
\begin{equation}\begin{aligned}
x_{n+1} &= \wrap(x_{n} + \inc(f_x + m_x \cos (2 \pi y_{n-d}))) \\
y_{n+1} &= \wrap(y_{n} + \inc(f_y + m_y \cos (2 \pi x_{n-d})))
\end{aligned}\label{eq:osc}\end{equation}
where
\begin{equation*}\wrap(t) = t - \lfloor t \rfloor ,\quad \inc(t) = \frac{440}{\SR} 2^{\frac{t - 69}{12}}\end{equation*}
Here $x_n$, $y_n$ is the phase of each oscillator at time step $n$,
$d$ is a delay measured in samples, $f_x$, $f_y$ is the base frequency
of each oscillator as a MIDI note number, and $m_x$, $m_y$ is the
modulation index of each oscillator as a MIDI note number.  $\wrap(t)$
performs wrapping into $[0,1)$, with $\lfloor t \rfloor$ being the
flooring operation (the largest integer not greater than $t$).

The four-dimensional parameter space vector will be written
\begin{equation*}a=(f_x,f_y,m_x,m_y)\end{equation*}
and the $(2d+2)$-dimensional phase space vector
\begin{equation*}z=(x_n,y_n,x_{n-1},y_{n-1},\ldots,x_{n-d},y_{n-d})\end{equation*}
with sample rate $\SR = 48000$.  For reasons
explained in Section~\ref{sec:glissues}, $d=1$ will be fixed.

\subsection{Lyapunov Exponents}

Lyapunov exponents can be used to measure the stability (or otherwise)
of a dynamical system.
A good introduction is found in Chapter~4.3 \emph{Lyapunov Exponent}~\cite{Elert07}.
The definition is covered in Chapter~13.7 \emph{Liapounov exponents and entropies}~\cite{Falconer03}
which also relates it to measures of fractal dimension.

The Lyapunov exponent $\lambda$ measures divergence in phase space:
\begin{equation*}\left| z_1(t) - z_0(t) \right| \approx e^{\lambda t} \left| z_1(0) - z_0(0) \right|\end{equation*}
\begin{equation}\lambda = \lim_{\begin{array}{c} t \to \infty \\
z_1(0) \to z_0(0)\end{array}} \frac{1}{t} \log{\frac{\left|z_1(t) - z_0(t)\right|}{\left|z_1(0) - z_0(0)\right|}}\label{eq:lexp}\end{equation}
An attracting orbit has $\lambda < 0$ and a divergent (chaotic) orbit
has $\lambda > 0$.

A modified norm is required to take into account
the wrapping of phase into $[0,1)$:
\begin{equation*}\left|z\right|_{\wrap} = \sqrt{\sum_{i} {\left(\min(\wrap(z_i), 1 - \wrap(z_i))\right)^2}}\end{equation*}
For example the distance between $0.1$ and $0.9$ is properly $0.2$ (not $0.8$)
because $0.1$ can be phase-unwrapped to $1.1$.

\subsection{Viewing Planes}
```

```
An image is 2D, which requires choosing a subset of the 4D parameter
space to visualize. Two particular planes were chosen:
\begin{equation}\begin{aligned}
A_+(a_0, r_0) &= a_0 + r_0 \left(\begin{array}{rr}
1 & 0 \\
\phantom{-}1 & 0 \\
0 & 1 \\
0 & \phantom{-}1 \end{array}\right)\left(\begin{array}{c}u \\
v \end{array}\right) \\
A_-(a_0, r_0) &= a_0 + r_0 \left(\begin{array}{rr}
 1 &  0 \\
-1 &  0 \\
 0 &  1 \\
 0 & -1 \end{array}\right)\left(\begin{array}{c}u \\
v \end{array}\right)\end{aligned}\label{eq:view}\end{equation}
where $(u,v)$ is the coordinates of the pixel, $a_0$ is the
centre of the view, and $r_0$ is the radius of the view.

These planes were chosen because they are simple, while still being
flexible enough to explore the whole 4D space.
The $A_+$ plane varies both oscillators in the same direction, while
the $A_-$ plane varies each oscillator in opposite directions. To
center on a particular target point $(f_x, f_y, m_x, m_y)$ one might
use the $A_+$ plane to center on the midpoint
\begin{equation*}\left(\frac{f_x + f_y}{2}, \frac{f_x + f_y}{2}, \frac{m_x + m_y
    ↳ }{2}, \frac{m_x + m_y}{2}\right)\end{equation*}
and then switch to the $A_-$ plane to break the $(x,y)$ symmetry.

\section{Implementation}

The implementation uses OpenGL~\cite{OpenGL} and OpenGL Shading Language~\cite{
    ↳ GLSL}
for computation and graphical rendering, GLUT~\cite{GLUT} for windowing
and input event handling, and JACK~\cite{JACK} for audio output.

\subsection{Introduction to Modern OpenGL}

Modern OpenGL has a programmable shader pipeline. Vertex attributes are read
from vertex buffers and processed by vertex shaders. The outputs
of the vertex shader (called varyings) are further manipulated by an
optional geometry shader stage. Geometry shaders can output a different
vertex count to their input count, whereas vertex shaders are one-in
one-out. The result of the geometry shader can be captured into another
vertex buffer using transform feedback. Following the geometry
shader the primitives (points or triangles) are rasterized, and
varyings interpolated across each primitive. Finally a fragment
shader takes these values and computes the colour at that pixel. The
output of a fragment shader can be captured by attaching a texture to
a framebuffer.

\subsection{Computation Overview}

To render an image a texture is first filled with $(u,v)$ coordinates
using a framebuffer object and a fragment shader. This texture is copied
to a vertex buffer object, interleaved with an initial phase space vector
$z=(0,0,0,0)$ and Lyapunov exponent statistics vector $l=(0,0,0,0)$ for
each point.
```

```
Using a vertex shader, $a$ is calculated from $(u,v)$ using Equation~\ref{eq:view}, and then
a rough estimate of the Lyapunov exponent is computed using
Equation~\ref{eq:lexp} by perturbing $z_1(0) = z_0(0) + \delta$ with
$\delta$ small and performing $t = 256$ iterations of Equation~\ref{eq:osc}.
The first few repetitions are discarded, along with those resulting in $-\infty$,
and the rough $\lambda$ estimates are accumulated in $l$.

Between each repetition the working set is compacted using a geometry
shader.  Points whose mean Lyapunov exponent estimate changed
very little during the previous step are plotted and removed from the
working set.  The other points are kept to be refined
further, directing the computational effort on the points that
need it most:~those slow to converge.

To ensure user interface responsiveness, the computation is amortized
over several frames.  The target frame period is divided by the measured
time for one repetition to compute how many repetitions to perform that
frame.  The repetitions-per-frame increases as the working set becomes
smaller.

\subsection{Noise Increases Stability}

At the end of each repetition $z_1$ is kept instead of $z_0$.  This
effectively adds a small amount of noise, counter-intuitively
increasing stability.  Noise allows more of the phase space to be
explored, and makes it more likely for the perturbed orbit to reach an
attracting part of the phase space.

\subsection{Dither Increases Quality}

To reduce grid sampling artifacts, $(u,v)$ is perturbed within the bounds
of its corresponding pixel before calculating the $a$ parameter vector
for each repetition.

\section{Results}

\begin{figure*}[ht!]
\centering
\begin{tabular}{cc}
\subfigure[$A_+((120, 120, 0, 0), 72)$]{\includegraphics[width=3in]{example01.png}\label{ex:01}} &
\subfigure[$A_-((120, 120, 0, 0), 72)$]{\includegraphics[width=3in]{example02.png}\label{ex:02}} \\
\subfigure[$A_+((95.2, 95.2, 32.6, 32.6), 4.5)$]{\includegraphics[width=3in]{example03.png}\label{ex:03}} &
\subfigure[$A_-((95.2, 95.2, 32.6, 32.6), 4.5)$]{\includegraphics[width=3in]{example04.png}\label{ex:04}} \\
\subfigure[$A_+((151.57, 151.57, 1.64, 1.64), 0.07)$]{\includegraphics[width=3in]{example05.png}\label{ex:05}} &
\subfigure[$A_-((151.57, 151.57, 1.64, 1.64), 0.07)$]{\includegraphics[width=3in]{example06.png}\label{ex:06}} \\
\subfigure[$A_-((117.0, 148.4, 20.4, 2.7), 1.8)$]{\includegraphics[width=3in]{example07.png}\label{ex:07}} &
\subfigure[$A_+((103.65, 108.41, 33.42, 10.93), 0.14)$]{\includegraphics[width=3
```

```
    ↪ in]{example09.png}\label{ex:09}}
\end{tabular}
\caption{Example images.  Darker shades are stable, lighter shades chaotic.}
\label{fig:examples1}
\end{figure*}
%\begin{figure}[ht]
%\centering
%\subfigure[$A_-((117.0, 148.4, 20.4, 2.7), 1.8)$]{\includegraphics[width=3in]{↙
    ↪ example07.png}\label{ex:07}}
%\subfigure[$A_-((141.46, 146.22, 22.76, 0.27), 0.14)$]{\includegraphics[width=3↙
    ↪ in]{example08.png}\label{ex:08}}
%\subfigure[$A_+((103.65, 108.41, 33.42, 10.93), 0.14)$]{\includegraphics[width↙
    ↪ =3in]{example09.png}\label{ex:09}}
%\subfigure[$A_-((89.8, 137.5, -17.5, -7.1), 3.7)$]{\includegraphics[width=3in]{↙
    ↪ example10.png}\label{ex:10}}
%\caption{More examples.}
%\label{fig:examples2}
%\end{figure}

\subsection{Examples}

Figure~\ref{ex:01} shows the initial view on starting the interactive
browser.  Low frequencies to the left are stable even at high
modulation index away from the central axis.  High frequencies to the
right become chaotic at progressively lower modulation index.
\subref{ex:02} shows the $A_-$ plane at the same location.
\subref{ex:03} shows bands alternating between stability and
chaos.  The bands become distorted and collapse as the modulation
index and frequency increase.  \subref{ex:04} shows its $A_-$
plane, bands become rings.
When the frequency is greatly increased, the shapes become more
intricate.  \subref{ex:05} exhibits spirals of stability, with
similar spirals in the $A_-$ plane in \subref{ex:06}.

When $f_x=f_y$ and $m_x=m_y$ the $A_+$ plane has mirror symmetry about its
horizontal axis, and the $A_-$ plane has two-fold rotational symmetry
about its centre.
Breaking the symmetry and setting $f_x \ne f_y$ or $m_x \ne m_y$ leads
to diverse forms.  In particular Figure~\ref{ex:09} has shapes that
resemble those of Lyapunov space images of the logistic map.

\subsection{Interactive Explorer}

The implementation is an interactive audio-visual explorer for the parameter
space of coupled FM oscillators.
Clicking with the mouse zooms the view about the clicked point.
The left button (or scroll up) zooms in, the right button (or scroll
down) zooms out, the middle button centers the view on the target point.
Pressing the TAB key toggles between the $A_+$ and $A_-$ planes in
Equation~\ref{eq:view}, and F11 toggles full screen operation.

While the GPU simulates and analyses one oscillator pair per pixel, the
CPU simulates one oscillator pair with $a$ determined from the pixel
under the mouse pointer.  The image acts as a map,
a reference frame for chosing parameters to audition by moving the mouse.

\section{Conclusions}
```

```
\subsection{Original Intent}

Earlier experiments used one Pure-data batch mode instance per CPU
core each sending analysis data to a realtime Pure-data instance.
The analysis used various methods (including FFT for spectral
statistics and the sigmund external for pitch tracking) to classify
points into pitched (ordered, stable) or unpitched (chaotic, unstable)
with measures of distortion or noisiness. Sadly this approach proved
impractical as it achieved only tens of pixels per second, even
with a fast multi-core CPU, and porting these signal analysis algorithms
to massively-parallel programmable graphics hardware seemed to be too
difficult.

\subsection{OpenGL Issues \label{sec:glissues}}

The current implementation is hardcoded with delay $d=1$ and would be
very awkward to generalize. OpenGL architecture limits each vertex
attribute to four components with the maximum number of attributes
typically limited to sixteen. This totals 64 floats per vertex, 6 of
which are needed for the pixel coordinates and Lyapunov exponent
statistics accumulation. Therefore using OpenGL imposes a limit
$d < 28$. For comparison the original experiments in
\emph{Soft Rock EP} used Pure-data's default block size of 64, with $d = 32$.
Moreover, increasing $d$ increases video memory consumption. With
the maximum $d = 27$, browsing at $1920 \times 1080$ resolution would
require over 1GB.

Future work on this project will look into using OpenCL, which
provides a heterogenous CPU and GPU computation framework, in the hope that
it will avoid the inherent awkwardness of abusing OpenGL shaders to
perform calculations.

\subsection{Audio Issues}

While the implementation works as intended, with $d=1$
the sound is nowhere near as rich and varied as with $d=32$. With
small $d$ there is much more very high frequency content in
interesting-looking regions. There seem to be few if any regions of the
parameter space with both interesting appearance and palatable audio
frequencies at $d=1$, while with high $d$ there are
parameters that generate sounds that fluctuate intermittently between
smooth tones and noise. Visualization with high $d$ has not been
possible so far, so whether their neighbourhoods
look as interesting as they sound remains an openquestion .

Unfortunately, heavy use of the GPU in the interactive browser can
block the operating system for too long and cause audible glitches
(JACK xruns). This situation may change as free drivers continue to
improve, allowing use of the browser in a live situation.

\subsection{Pretty Pictures}

Despite these shortcomings, I think the images look good.
I plan to render a selection at high resolution and print
postcards and posters. For huge images it is possible to divide the image plane↙
    ↳ into tiles and
```

```
compute each tile in succession, finally combining the pieces
into one large picture.

There is also scope for video work, moving and rotating the viewing
plane through the 4D parameter space, with different shapes forming
and collapsing over time.  Rough benchmarks take 5-10 seconds
per frame at $1920 \times 1080$, so it seems sensible to wait until faster ↙
    ↳ cheaper
graphics cards become available.

\section{Obtaining the Implementation}

The implementation was written on GNU/Linux Debian Wheezy running on a quad-core↙
    ↳ AMD64 processor
with NVIDIA GTX~550Ti graphics card using
proprietary drivers.  The source code is available at:
\url{http://code.mathr.co.uk/lyapunov-fm}


\section{Acknowledgements}

Thanks to the anonymous reviewers for their constructive criticism
on a number of issues, and to Rob Canning, Adnan Hadzi, and
Joanne Seale for their helpful feedback on earlier versions of this
paper.

\bibliographystyle{acl}
\bibliography{lyapunov-fm}

\end{document}
```

7 doc/Makefile

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PAPER=lyapunov-fm

all: $(PAPER).pdf

clean: cleantmp
	rm -f $(PAPER).pdf

cleantmp:
	rm -f $(PAPER).aux
	rm -f $(PAPER).bbl
	rm -f $(PAPER).blg
	rm -f $(PAPER).log

%.pdf: %.tex %.bib
	pdflatex "$<"
	bibtex `basename "$<" .tex`
	pdflatex "$<"
	pdflatex "$<"

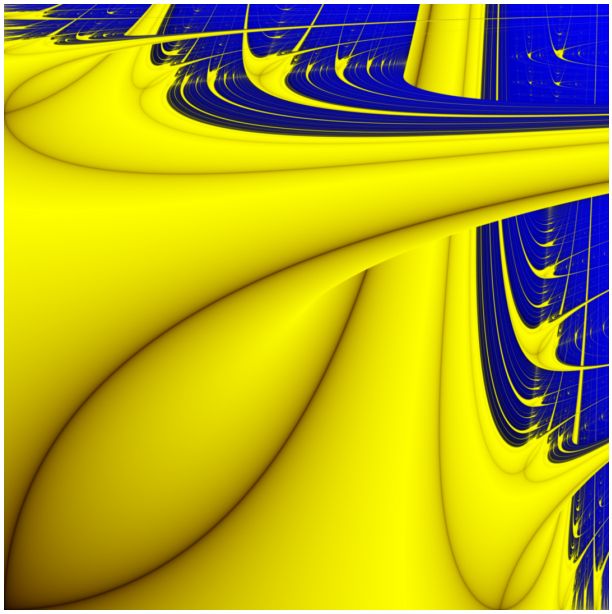
.PHONY: all clean
```

8 doc/slides/bifurcation2d.c

```c
#include <math.h>
#include <stdio.h>
#include <stdlib.h>

#define W (1080*4)
#define H (1080*4)
#define N 2500
#define A 1

int main(int argc, char **argv) {
  float lmin =  1000;
  float lmax = -1000;
  printf("P6\n%d %d\n255\n", W, H);
  for (int y = H - 1; y >= 0; --y) {
    for (int x = 0; x < W; ++x) {
      float l = 0;
      int n = 0;
      for (int dx = 0; dx < A; ++ dx) {
        for (int dy = 0; dy < A; ++ dy) {
          float rB = 2 + (x + dx / (float) A) * 2 / W;
          float rA = 2 + (y + dy / (float) A) * 2 / H;
          float z = 0.5;
          for (int i = 0; i < 250; ++i) {
            z = rA * z * (1 - z);
            z = rB * z * (1 - z);
          }
          for (int i = 0; i < N; ++i) {
            z = rA * z * (1 - z);
            float dz = fabsf(rA * (1 - 2 * z));
            if (dz > 0) {
              l += logf(dz);
              n += 1;
            }
            z = rB * z * (1 - z);
            dz = fabsf(rB * (1 - 2 * z));
            if (dz > 0) {
              l += logf(dz);
              n += 1;
            }
          }
        }
      }
      float v = -100;
      if (n != 0) {
        l /= n;
        v = l;
        lmin = fminf(lmin, l);
        lmax = fmaxf(lmax, l);
      }
      float fr, fg, fb;
      if (v <= 0) {
        fr = (1 + tanhf(v / 4)) * 255;
        fg = (1 + tanhf(v / 3)) * 255;
        fb = 0;
      } else {
        fr = 0;
        fg = 0;
```

```
        fb = tanhf(3 * v) * 255;
      }
/*
      int cr = fminf(fmaxf((255 * 0.6 * (1.0 - tanhf(4.0 * (v - 0.3333))) - 64) ↙
          ↳ * 255.0 / 191, 0), 255);
      int cg = fminf(fmaxf((255 * (2.0 * v * v - 2.5 * v + 1) - 64) * 255.0 / ↙
          ↳ 191, 0), 255);
      int cb = fminf(fmaxf((255 * v * v - 64) * 255.0 / 191, 0), 255);
*/
#define O(c) putchar(fminf(fmaxf(c, 0), 255))
      O(fr);O(fg);O(fb);
    }
  }
  fprintf(stderr, "%f %f\n", lmin, lmax);
  return 0;
}
```

9 doc/slides/bifurcation-2d.png

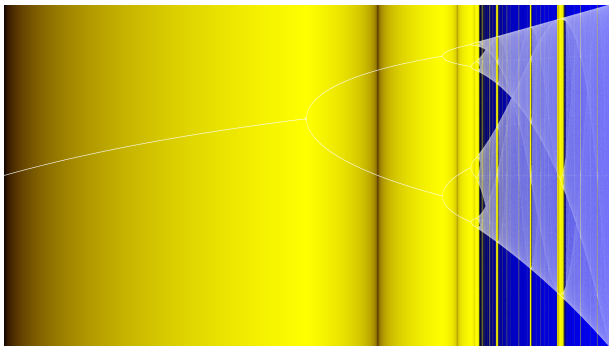


10 doc/slides/bifurcation.c

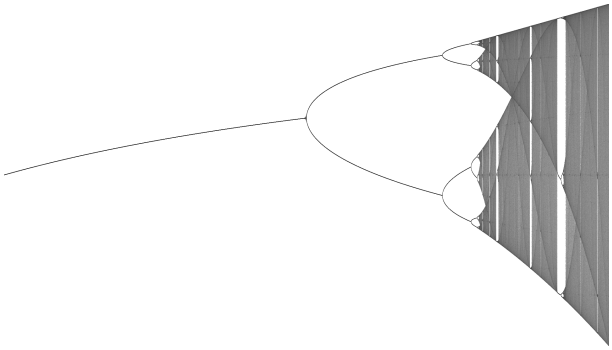
```
#include <math.h>
#include <stdio.h>
#include <stdlib.h>

#define W 1920
#define H 1080
#define N 50000
#define A 16

int main(int argc, char **argv) {
  int *m = calloc(1, W * H * sizeof(int));
  unsigned char *grey = calloc(1, W * H);
  unsigned char *rgb = calloc(1, W * H * 3);
```

```
  for (int x = 0; x < W; ++x) {
    float l = 0;
    int n = 0;
    for (int dx = 0; dx < A; ++ dx) {
      float r = 2 + (x + dx / (float) A) * 2 / W;
      float z = 0.5;
      for (int i = 0; i < 500; ++i) {
        z = r * z * (1 - z);
      }
      for (int i = 0; i < N; ++i) {
        z = r * z * (1 - z);
        float dz = fabsf(r * (1 - 2 * z));
        if (dz > 0) {
          l += logf(dz);
          n += 1;
        }
        float dither = rand() / (float) RAND_MAX - 0.5f;
        int y = fminf(fmaxf(H * (1 - z) + dither, 0), H - 1);
        m[y * W + x]++;
      }
    }
    if (n != 0) {
      l /= n;
    } else {
      l = -100;
    }
    float v = 0.5 + 0.5 * tanh(3.0 * l);
    int cr = fminf(fmaxf((255 * 0.6 * (1.0 - tanh(4.0 * (v - 0.3333))) - 30) * ↙
        ↳ 255.0 / 225, 0), 255);
    int cg = fminf(fmaxf((255 * (2.0 * v * v - 2.5 * v + 1) - 30) * 255.0 / 225,↙
        ↳  0), 255);
    int cb = fminf(fmaxf((255 * v * v - 30) * 255.0 / 225, 0), 255);
    for (int y = 0; y < H; ++ y) {
      float ca = log2f(1 + m[y * W + x]) / log2f(1 + A * N);
      grey[y * W + x] = 255 * (1 - ca);
      rgb[3 * (y * W + x) + 0] = 255 * ca + (1 - ca) * cr;
      rgb[3 * (y * W + x) + 1] = 255 * ca + (1 - ca) * cg;
      rgb[3 * (y * W + x) + 2] = 255 * ca + (1 - ca) * cb;
    }
  }
  printf("P5\n%d %d\n255\n", W, H);
  fwrite(grey, W * H, 1, stdout);
  printf("P6\n%d %d\n255\n", W, H);
  fwrite(rgb, W * H * 3, 1, stdout);
  return 0;
}
```

11 doc/slides/bifurcation-colour.png



12 doc/slides/bifurcation-grey.png



13 doc/slides/colourize.c

```
#include <math.h>
#include <stdio.h>
#include <stdlib.h>

int main(int argc, char **argv) {
  int count = 4096 * 4096;
  unsigned short *pgm = malloc(count * 2);
  FILE *f = fopen(argv[1], "rb");
  fseek(f, -count * 2, SEEK_END);
  fread(pgm, count * 2, 1, f);
  fclose(f);
  unsigned short mi = 65535, ma = 0;
  for (int i = 0; i < count; ++i) {
    if (mi > pgm[i]) mi = pgm[i];
    if (ma < pgm[i]) ma = pgm[i];
  }
  unsigned char *ppm = malloc(count * 3);
  int k = 0;
  for (int i = 0; i < count; ++i) {
    float g = pgm[i];
    if (g > 65536/2) {
      g -= 65536/2;
      g /= (ma - 65536/2);
```

```
      g *= 0.5;
      g += 0.5;
    } else {
      g -= mi;
      g /= (65536/2 - mi);
      g *= 0.5;
    }
    float x = 0.6 * (1.0 - tanh(4.0 * (g - 1.0/3.0)));
    float y = 2.0 * g * g - 2.5 * g + 1.0;
    float z = g * g;
    x -= 0.25; x /= 0.75; x = fmin(fmax(x, 0), 1); x = pow(x, 0.25);
    y -= 0.25, y /= 0.75; y = fmin(fmax(y, 0), 1); y = pow(y, 0.25);
    z -= 0.25, z /= 0.75; z = fmin(fmax(z, 0), 1); z = pow(z, 0.25);
    ppm[k++] = 255 * x;
    ppm[k++] = 255 * y;
    ppm[k++] = 255 * z;
  }
  f = fopen(argv[2], "wb");
  fprintf(f, "P6\n4096 4096\n255\n");
  fwrite(ppm, count * 3, 1, f);
  fclose(f);
  return 0;
}
```

14 doc/slides/inc-equation.png

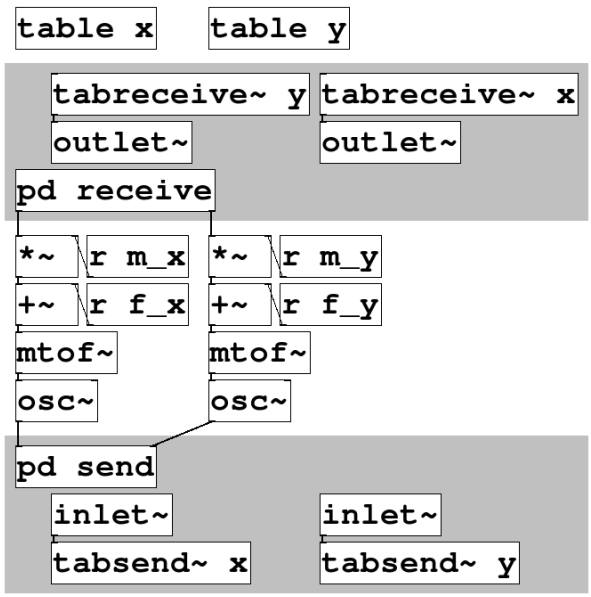
$$\operatorname{inc}(n) = \frac{440}{\mathrm{SR}} 2^{\frac{n-69}{12}}$$

15 doc/slides/maths.tex

```
\documentclass[11pt,a4paper]{article}
\usepackage{amsmath}
\usepackage{mathtools}

\begin{document}

\[
\begin{aligned}
\begin{matrix*}[r]
x_{j+1} \\
y_{j+1}
\end{matrix*} &=
\operatorname{wrap}\left(
\begin{matrix*}[r]
x_j \\
y_j
\end{matrix*} +
\operatorname{inc}\left(
\begin{matrix*}[r]
f_x \\
f_y
```

```
\end{matrix*} +
\begin{matrix*}[r]
m_x \\
m_y
\end{matrix*} \cos\left(2 \pi
\begin{matrix*}[r]
y_{j-d} \\
x_{j-d}
\end{matrix*} \right) \right) \right)
\\
\operatorname{inc}\left(n\right) &=
\mbox{\small$\frac{440}{\operatorname{SR}}$} 2^{\mbox{\Large$\frac{n - 69}{12}$}}
\end{aligned}\\
\mbox{\Huge$\infty$}
\]

\end{document}
```

16 doc/slides/puredata.png



17 doc/slides/recurrence-equation.png

$$\begin{matrix} x_{j+1} \\ y_{j+1} \end{matrix} = \operatorname{wrap}\left(\begin{matrix} x_j \\ y_j \end{matrix} + \operatorname{inc}\left(\begin{matrix} f_x \\ f_y \end{matrix} + \begin{matrix} m_x \\ m_y \end{matrix} \cos\left(2\pi \begin{matrix} y_{j-d} \\ x_{j-d} \end{matrix}\right)\right)\right)$$

18 doc/slides/slides.hs

```
{-# LANGUAGE NoMonomorphismRestriction #-}
module Main (main) where

import Diagrams.Prelude
```

```
import Diagrams.Backend.Cairo.CmdLine (defaultMain)

import Control.Monad (forM_)
import Data.List (intersperse)
import Data.Maybe (fromJust)
import System.Environment (withArgs)

main = do
  forM_ (diagrams slides `zip` [100..]) $ \(d, i) ->
    withArgs ["-o", "o/" ++ tail (show i ++ ".png"), "-h", "1080"] $
      defaultMain d

diagrams = map style . presentation

slides =
  ( "Lyapunov FM"
  ,[( "Background"
     ,[( "Recurrence Relations"
       ,[( "Recurrence"
         , [ "Next state depends on current state."
           , "z(t+1) = F(z(t))"
       ]),("Phase Space"
         , [ "Internal state that varies over time."
           , "z(0) determines all future z(t)."
       ]),("Parameter Space"
         , [ "Constant over time."
           , "Family of recurrences."
       ]),("Example: Logistic Map"
         , [ "z(t+1) = r z(t) (1 - z(t))"
           , "1D phase space: z(t)"
           , "1D parameter space: r"
       ])]
     ),( "Behaviour Of Recurrences"
       ,[( "Orbits"
         , [ "The set of points reached from z(0)."
           , "{ z(0), z(1), z(2), ... }"
       ]),("Attractors"
         , [ "Orbits may become periodic."
           , "Many points can reach one attractor."
       ]),("Stability"
         , [ "Nearby points are pulled closer together."
       ]),("Chaos"
         , [ "Nearby points are pushed further apart."
           , "Butterfly effect."
       ]),("Lyapunov Exponent"
         , [ "Quantifies butterfly effect."
           , "< 0 : stable"
           , "> 0 : chaotic"
       ])]
     ),( "Mapping Behaviours"
       ,[( "1D Logistic Map"
         , [ "z(t+1) = F(z(t))"
           , "F(x) = r x (1 - x)"
           , "1D phase space: z(t)"
           , "1D parameter space: r"
       ]),("1D Bifurcation Diagram"
         , [ "#LogisticBifurcation"
```

```
        ]),("1D Lyapunov Space"
          , [ "#LogisticLyapunov1D"
        ]),("2D Logistic Map"
          , [ "z(t+1) = G(F(z(t)))"
            , "F(x) = A x (1 - x)"
            , "G(x) = B x (1 - x)"
            , "1D phase space: z(t)"
            , "2D parameter space: (A,B)"
        ]),("2D Lyapunov Space"
          , [ "#LogisticLyapunov2D"
        ])])
    ]),("Implementation"
      ,[( "Coupled FM Oscillators"
        ,[( "Pure-data Implementation"
          , [ "#Pure-data"
        ]),("Recurrence Relation"
          , [ "x(t+1) = F(fx, mx, x(t), y(t-d))"
            , "y(t+1) = F(fy, my, y(t), x(t-d))"
            , "F(f,m,u,v) = wrap(u + I(f + m cos(2 pi v)))"
            , "I(n) = 440/SR 2^((n - 69)/12) "
        ]),("4D Parameter Space"
          , [ "Base frequency (oscillator X)"
            , "Base frequency (oscillator Y)"
            , "Modulation index (oscillator X)"
            , "Modulation index (oscillator Y)"
        ])]
      ),( "GPU Overview"
        ,[( "OpenGL Pipeline"
          , [ "#OpenGLPipeline"
        ]),("OpenGL Glossary"
          , [ "shader: stage in the pipeline"
            , "program: a linked set of shaders"
            , "uniform: parameters for a program"
            , "attribute: per-vertex shader input"
            , "varying: data passed between shaders"
            , "texture: 2D array stored in GPU memory"
            , "buffer: 1D array stored in GPU memory"
        ]),("Hardware Limits"
          , [ "16 attributes"
            , "4 components per attribute"
            , "2(d+1) components for phase space vector"
            , "5 components needed elsewhere"
            , "d <= 28 with optimal packing"
        ]),("Buffer Memory Layout"
          , [ "#Memory"
        ]),("Vertex Attribute Pointers"
          , [ "#Pointers"
        ])]
      ),( "GPU Details"
        ,[( "Computation Structure"
          , [ "#Compute"
        ]),( "Fill RGBA Texture with uv00"
          , [ "#Fill"
        ]),("Copy RG Channels to Buffer"
          , [ "#Copy"
        ]),("Initialize State"
          , [ "#Init"
```

```
    --       ]),("Step: Unpack State"
120 --         , ["#Unpack"
           ]),("Step Recurrence"
             , ["#Step"
    --       ]),("Step: Pack State"
    --         , ["#Pack"
125        ]),("Prune Working Set"
             , ["Memory bandwidth limits speed"
               , "Some points converge faster"
               , "Geometry shader culls converged points"
               , "Working set becomes leaner"
130        ]),("Plot Points"
             , ["Render to framebuffer texture"
               , "gl_FragColor = raw L statistics"
           ]),("Colour Image"
             , ["Read raw L statistics from texture"
135            , "gl_FragColor = F(1)"
               , "stable -> yellow"
               , "chaotic -> blue"
           ])]
       ),("Lyapunov Exponents"
140      ,[("Informal Definition"
             , ["|Delta z(t)| = exp(L t) |Delta z(0)|"
               , "L < 0 stable; nearby values pulled closer"
               , "L > 0 chaotic; nearby values pushed apart"
           ]),("Formal Definition"
145        , ["L = 1/t log |Delta z(t)|/|Delta z(0)|"
             , "Take limits as t -> infinity and Delta z(0) -> 0"
           ]),("Calculation in 1D"
             , ["Inner limit become derivative"
               , "Recurrence derivative is product of step derivatives"
150            , "Log of product is sum of logs"
           ]),("Calculation in >1D"
             , ["Inner limit becomes Jacobian"
               , "Outer limit defines a matrix"
               , "Log of matrix eigenvalues are Lyapunov spectrum"
155            , "Maximal Lyapunov exponent measures stability"
           ]),("Numerical Calculation"
             , ["Pick a point on the attractor z(0)"
               , "Pick a nearby point w(0)"
               , "Run recurrence on both until t large"
160            , "Compute L = 1/t log |z(t)-w(t)|/|z(0)-w(0)|"
               , "Repeat, averaging all the results"
           ])])
     ]),("Conclusions"
        ,[("Examples"
165        ,[("d = 0"
             , ["#Example0"
           ]),("d = 1"
             , ["#Example1"
           ]),("d = 4"
170          , ["#Example4"
           ]),("d = 16"
             , ["#Example16"
           ])]
       ),("Evaluation"
175      ,[("Good Points"
```

```
          , [ "Operation at 0 <= d <= 27"
            , "Experimental fractional d (interpolation)"
            , "Batch mode for offline rendering"
            , "Tiled rendering of huge images"
        ]),("Bad Points"
          , [ "JACK xruns with heavy GPU load"
            , "Free drivers might be better than proprietary?"
            , "Perceptual (ir)relevance of Lyapunov exponent"
            , "Tiled rendering random seed glitches"
            , "Long video render times even with fast hardware"
        ])]
      ),( "Ends"
        ,[( "Links"
          , [ "http://mathr.co.uk/lyapunov-fm"
            , "claude@mathr.co.uk"
  ])])])])

images =
  [("LogisticBifurcation", d'bifur)
  ,("LogisticLyapunov1D", d'bifurc)
  ,("LogisticLyapunov2D", d'bifurc2)
  ,("Pure-data", d'pd)
  ,("OpenGLPipeline", d'opengl)
  ,("Compute", d'compute)
  ,("Memory", d'memory)
  ,("Pointers", d'pointers)
  ,("Fill", d'fill)
  ,("Copy", d'copy)
  ,("Init", d'init)
  ,("Unpack", d'unpack)
  ,("Step", d'step)
  ,("Pack", d'pack)
  ,("Example0", hcat [image "0_pos.png" 9 9, strutX 1, image "0_neg.png" 9 9] # centerXY)
  ,("Example1", hcat [image "1_pos.png" 9 9, strutX 1, image "1_neg.png" 9 9] # centerXY)
  ,("Example4", hcat [image "4_pos.png" 9 9, strutX 1, image "4_neg.png" 9 9] # centerXY)
  ,("Example16", hcat [image "16_pos.png" 9 9, strutX 1, image "16_neg.png" 9 9] # centerXY)
  ]

presentation (t, ss) = ([], mempty) : ([t], bullets (map fst ss)) : concatMap (section [t]) ss

section ts (t, ss) = (t:ts, bullets (map fst ss)) : concatMap (subsection (t:ts)) ss

subsection ts (t, ss) = (t:ts, bullets (map fst ss)) : concatMap (slide (t:ts)) ss

slide ts (t, ss) = [(t:ts, bullets ss)]

bullet ('#':im) = fromJust $ im `lookup` images
bullet s = llabel s (0, 0) # scale 1.5 # translate (r2(-11.5, 0))
```

```
225 bullets ss = (vcat . intersperse (llabel "" (0,0)) . map bullet) ss # alignT # ↙
      ↳ translate (r2(0, 3))

    style (ts, d)
      = (d <> (frame ts # centerXY))
      # font "LMSans10"
230   # lw 0.05
      # lc black
      # lineCap LineCapRound
      # lineJoin LineJoinRound
      # fc white
235   # centerXY
    -- # pad 1.1
      # bg white

    frame [] = mconcat
240   [ alignedText 0 0.5 "Lyapunov Space of" # scale 1.5 # translate (r2(0.5, 14)) ↙
        ↳ # fc black # bold
    , alignedText 0 0.5 "Coupled FM Oscillators" # scale 1.5 # translate (r2(0.5, ↙
        ↳ 12.5)) # fc black # bold
    , alignedText 0 0.5 "Claude Heiland-Allen" # scale 1.5 # translate (r2(0.5, ↙
        ↳ 10)) # fc black
    , alignedText 0 0.5 "Linux Audio Conference 2013" # scale 1.5 # translate (r2↙
        ↳ (0.5, 7.5)) # fc black
    , box (0, 0) (24, 18)
245   ]
    frame (title:ts) = mconcat $
      [ alignedText 0 0.5 t # scale 0.5 # translate (r2(0.5, 17.25 - i))# fc black |↙
        ↳ (t, i) <- reverse ts `zip` [0..] ] ++
      [ alignedText 0 0.5 title # scale 1.5 # translate (r2(0.5, 14)) # fc black # ↙
        ↳ bold
    , alignedText 0 0.5 "Claude Heiland-Allen" # scale 0.5 # translate (r2(0.5, ↙
        ↳ 0.75)) # fc black
250   , alignedText 1 0.5 "Linux Audio Conference 2013" # scale 0.5 # translate (r2↙
        ↳ (23.5, 0.75)) # fc black
    , box (0, 0) (24, 18)
      ]

    label s p = bound (text s # scale 0.75) # translate (r2 p) # fc black
255
    llabel s p = bound (alignedText 0 0.5 s # scale 0.75) # translate (r2 p) # fc ↙
      ↳ black

    rlabel s p = bound (alignedText 1 0.5 s # scale 0.75) # translate (r2 p) # fc ↙
      ↳ black

260 bound d = d `atop` phantom' (square 0.5)

    phantom' d = phantom d `asTypeOf` d

    withEnvelope' d d' = withEnvelope (d `asTypeOf` d') d'
265
    line p q = fromVertices [p2 p, p2 q]

    triangle p q r = fromVertices [p2 p, p2 q, p2 r, p2 p] # fc black

270 box (x0, y0) (x1, y1) = mconcat
```

```
    [ line (x0, y0) (x1, y0)
    , line (x1, y0) (x1, y1)
    , line (x1, y1) (x0, y1)
    , line (x0, y1) (x0, y0)
    ]

xarrow (x0, y0) (x1, y1) = mconcat
    [ line (x0, y0) (x1 - 0.25, y1)
    , triangle (x1, y1) (x1 - 0.5, y1 + 0.25) (x1 - 0.5, y1 - 0.25)
    ]

yarrow (x0, y0) (x1, y1) = mconcat
    [ line (x0, y0) (x1, y1 - 0.25)
    , triangle (x1, y1) (x1 - 0.25, y1 - 0.5) (x1 + 0.25, y1 - 0.5)
    ]

nxarrow (x0, y0) (x1, y1) = mconcat
    [ line (x0, y0) (x1 + 0.25, y1)
    , triangle (x1, y1) (x1 + 0.5, y1 + 0.25) (x1 + 0.5, y1 - 0.25)
    ]

nyarrow (x0, y0) (x1, y1) = mconcat
    [ line (x0, y0) (x1, y1 + 0.25)
    , triangle (x1, y1) (x1 - 0.25, y1 + 0.5) (x1 + 0.25, y1 + 0.5)
    ]

d'fill = mconcat
    [ label "u" (9, 3.5)
    , label "v" (1.5, 8.5)
    , label "R" (17.5, 11.5)
    , label "G" (18.5, 11.5)
    , label "B" (19.5, 11.5)
    , label "A" (20.5, 11.5)
    , label "u" (17.5, 10.5), box (17, 10) (18, 11)
    , label "v" (18.5, 10.5), box (18, 10) (19, 11)
    , label "0" (19.5, 10.5), box (19, 10) (20, 11)
    , label "0" (20.5, 10.5), box (20, 10) (21, 11)
    , yarrow (2, 5) (2, 12)
    , xarrow (3, 4) (15, 4)
    , box (3, 5) (15, 12)
    , box (8, 7) (8.5, 7.5)
    , line (8, 7.5) (17, 11)
    , line (8.5, 7) (21, 10)
    ] # centerXY

d'copy = grid
    [ [ cell "11", cell "21", hdots, cell "w1" ]
    , [ cell "12", cell "22", hdots, cell "w2" ]
    , [ vdots', vdots', ddots, vdots' ]
    , [ cell "1h", cell "2h", hdots, cell "wh" ]
    ] # centerXY
  ==== strutY 1 ==== nyarrow (0, 2) (0, 0) ==== strutY 1 ====
  hcat [cell' "11", cell' "21", hdots, cell' "w1", cell' "12", hdots, cell' "w2
      ↪ ", cell' "13", hdots, cell' "wh" ] # centerXY
  where vdots' = centerXY vdots <> phantom' (centerXY (box (0,0) (4,2)))

```

```
hdots = mconcat [ dot (0.5, 0.5), dot (1, 0.5), dot (1.5, 0.5), phantom' (box ↙
    ↳ (0, 0) (2, 1)) ]

vdots = mconcat [ dot (0.5, 0.5), dot (0.5, 1), dot (0.5, 1.5), phantom' (box ↙
    ↳ (0, 0) (1, 2)) ]

ddots = mconcat [ dot (0.5, 1.5), dot (1, 1), dot (1.5, 0.5), phantom' (box (0, ↙
    ↳ 0) (2, 2)) ]

dot p = circle 0.1 # fc black # translate (r2 p)

cell [x,y] = hcat [sub ['u',x,y], sub ['v',x,y], sub "0", sub "0"]

cell' [x,y] = hcat [sub ['u',x,y], sub ['v',x,y]]

chunk _ [] = []
chunk n xs = let (ys, zs) = splitAt n xs in ys : chunk n zs

grid = vcat . map (centerXY . hcat . map centerXY)

d'init = ((vcat
  [ label "in" (6, 0.5)
  , strutY 7 # translate (r2(6,0))
  , vec2 "q" "u" "v"
  ] # centerXY) ||| strutX 1 ||| xarrow (0,0) (2, 0) ||| strutX 1 ||| (vcat
  [ label "out" (6, 0.5)
  , strutY 1 # translate (r2(6,0))
  , vec4' "p0" "0" "0" "0" "0"
  , vec4' "p1" "0" "0" "0" "0"
  , vdots # translate (r2(7.5, 0))
  , vec4' "pN" "0" "0" "0" "0"
  , vec4 "l" "0" "0" "0" "0"
  , vec2 "q" "u" "v"
  ] # centerXY)) # centerXY

d'unpack = ((vcat
  [ label "in" (6, 0.5)
  , strutY 2.5 # translate (r2(6,0))
  , vec4' "p0" "x0" "y0" "x1" "y1"
  , strutY 0.5 # translate (r2(6,0))
  , vdots # translate (r2(7.5, 0))
  , strutY 0.5 # translate (r2(6,0))
  , vec4' "pN" "xd-1" "yd-1" "xd" "yd"
  , strutY 0.5 # translate (r2(6,0))
  ] # centerXY) ||| strutX 1 ||| xarrow (0,0) (2, 0) ||| strutX 1 ||| (vcat
  [ label "out" (6, 0.5)
  , strutY 1 # translate (r2(6,0))
  , float "j" "0"
  , vec2A "p[D]" "x0" "y0"
  , vec2A'       "x1" "y1"
  , vdots # translate (r2(7.5, 0))
  , vec2A'       "xd-1" "yd-1"
  , vec2A'       "xd" "yd"
  ] # centerXY) # centerXY === strutY 2 === label "" (0, 0.5)) # centerXY

d'pack = ((vcat
  [ label "in" (6, 0.5)
```

```
    , strutY 1 # translate (r2(6,0))
    , float "j" "j"
    , vec2A "p[D]" "x0" "y0"
    , vec2A'       "x1" "y1"
    , vdots # translate (r2(7.5, 0))
    , vec2A'       "xd-1" "yd-1"
    , vec2A'       "xd" "yd"
    ] # centerXY) ||| strutX 1 ||| xarrow (0,0) (2, 0) ||| strutX 1 ||| (vcat
    [ label "out" (6, 0.5)
    , strutY 2.5 # translate (r2(6,0))
    , vec4' "p0" "xj" "yj" "xj+1" "yj+1"
    , strutY 0.5 # translate (r2(6,0))
    , vdots # translate (r2(7.5, 0))
    , strutY 0.5 # translate (r2(6,0))
    , vec4' "pN" "xj-2" "yj-2" "xj-1" "yj-1"
    , strutY 0.5 # translate (r2(6,0))
    ] # centerXY)) # centerXY === strutY 2 ===
    label "subscripts are wrapped into [0,D)" (0,0.5)

memrowlayout = memrowvec

memrowvec = hcat $
  map subV4 ["p0", "p1"] ++ [hdots, subV4 "pN", subV4 "1", subV2 "q"]

memrowodd = hcat $
  map sub ["x0", "y0", "x1", "y1", "x2", "y2", "x3", "y3"] ++
  [hdots] ++ map sub ["xd-1", "yd-1", "xd", "yd"] ++
  map ub ["1", "L", "LL", "", "u", "v"]

memroweven = hcat $
  map sub ["x0", "y0", "x1", "y1", "x2", "y2", "x3", "y3"] ++
  [hdots] ++ map sub ["xd", "yd", "  ", "  "] ++
  map ub ["1", "L", "LL", "", "u", "v"]

subV4 v = label' v (2, 0.5) <> box (0,0) (4,1)

subV2 v = label' v (1, 0.5) <> box (0,0) (2,1)

sub v = label' v (0.5, 0.5) <> box (0,0) (1,1)

ub v = label v (0.5, 0.5) <> box (0,0) (1,1)

float v x = mconcat
  [ label "float" (4, 0.5), phantom' (box (3, 0) (5, 1))
  , label v (6, 0.5)
  , label x (7.5, 0.5), box (7, 0) (8, 1)
  ]

vec2 v x y = mconcat
  [ label "vec2" (4, 0.5), phantom' (box (3, 0) (5, 1))
  , label v (5.5, 0.5)
  , label x (6.5, 0.5), box (6, 0) (7, 1)
  , label y (7.5, 0.5), box (7, 0) (8, 1)
  ]

vec2A v x y = mconcat
  [ label "vec2" (4, 0.5), phantom' (box (3, 0) (5, 1))
```

```
    , label v (6, 0.5)
    , label' x (7.5, 0.5), box (7, 0) (8, 1)
    , label' y (8.5, 0.5), box (8, 0) (9, 1)
440 ]

vec2A' x y = mconcat
  [ phantom' (box (3, 0) (5, 1))
  , label' x (7.5, 0.5), box (7, 0) (8, 1)
445 , label' y (8.5, 0.5), box (8, 0) (9, 1)
  ]

vec4 v x y z w = mconcat
  [ label "vec4" (4, 0.5), phantom' (box (3, 0) (5, 1))
450 , label v (5.5, 0.5)
  , label x (6.5, 0.5), box (6, 0) (7, 1)
  , label y (7.5, 0.5), box (7, 0) (8, 1)
  , label z (8.5, 0.5), box (8, 0) (9, 1)
  , label w (9.5, 0.5), box (9, 0) (10, 1)
455 ]

label' [s1] (x, y) = label [s1] (x, y)
label' (s1:s2) (x, y) = mconcat [label [s1] (x - 0.1875, y), label s2 (2 * x + ↙
    ↳ 0.5, 2 * y - 0.5) # scale 0.5]

460 label'' [s1] (x, y) = label [s1] (x, y)
label'' (s1:s2) (x, y) = mconcat [label [s1] (x, y), llabel s2 (2 * x + 0.5, 2 *↙
    ↳  y - 0.5) # scale 0.5]

vec4' v x y z w = mconcat
  [ label "vec4" (4, 0.5), phantom' (box (3, 0) (5, 1))
465 , label' v (5.5, 0.5)
  , label' x (6.5, 0.5), box (6, 0) (7, 1)
  , label' y (7.5, 0.5), box (7, 0) (8, 1)
  , label' z (8.5, 0.5), box (8, 0) (9, 1)
  , label' w (9.5, 0.5), box (9, 0) (10, 1)
470 ]

d'memory = vcat
  [ memrowvec # centerXY
  , strutY 1
475 , text "d odd" # scale 0.75 # fc black <> strutY 1
  , strutY 0.5
  , memrowodd # centerXY
  , strutY 1
  , text "d even" # scale 0.75 # fc black <> strutY 1
480 , strutY 0.5
  , memroweven # centerXY
  ]

d'pointers = vcat
485 [ label "stride" (12, 0.5) <> phantom' (box (0,0) (1,1))
  , xarrow (2, 0.5) (22, 0.5) <> (withEnvelope' (box (0,0) (1,1)) $ (line (2, ↙
    ↳ 0.5) (2, 0) <> line (22, 0.5) (22, 0)) # dashed)
  , memrowlayout # translate (r2(2, 0))
  , pointer "p0" 2 1
  , pointer "p1" 6 2
490 , vdots <> phantom' (box (0,0) (1,1))
```

```
    , pointer "pN" 12 5
    , pointer "l"  16 6
    , pointer "q"  20 7
    ] # centerXY

pointer l x y = withEnvelope' p (p <> q)
  where
    p = label' l (0.5, 0.5) <> xarrow (1, 0.5) (x, 0.5) <> phantom' (box (0,0) ↙
        ↳ (1,1))
    q = line (x, 0.5) (x, y) # dashed

d'step = vcat
  [ hcat [hdots, v "j-d-1", v "j-d", v "j-d+1", hdots, v "j-1", v "j", v "j+1", ↙
      ↳ v "j+2", hdots]
  , mconcat
      [ line (5, 2) (5, 1)
      , line (13, 2) (13, 1)
      , line (5, 1) (13, 1)
      , line (9, 1) (9, 0)
      , line (9, 0) (15, 0)
      , yarrow (15, 0) (15, 2)
      ]
  ] # centerXY === strutY 1 ===
  image "recurrence-equation.png" 16 2
  === strutY 0.5 ===
  image "inc-equation.png" 8 2

v' [a,b] s = label'' ([a]++s) (0.5, 1) <> label'' ([b]++s) (0.5, 0) <> phantom' ↙
    ↳ (box (0,-0.5) (2,1.5))

v s = v' "xy" s <> box (0,-0.5) (2,1.5)

d'pd = image "puredata.png" 9 9

d'opengl = mconcat
  [ label "vertex attributes" (-1, 1)
  , nyarrow (0, 0) (0, -2), llabel "vertex shader" (1, -1)
  , nyarrow (0, -3) (0, -5), llabel "geometry shader" (1, -4)
  , line (-0.5, -5.5) (-2, -5.5), yarrow (-2, -5.5) (-2, 0), rlabel "transform ↙
      ↳ feedback" (-3, -2)
  , nyarrow (0, -6) (0, -8), llabel "fragment shader" (1, -7)
  , label "fragment colour" (-1, -9)
  ]

d'compute = mconcat
  [ label "fill" (1, 6)
  , label "copy" (4, 6)
  , label "init" (7, 6)
  , label "colour" (4, 1)
  , label "plot" (7, 1)
  , label "step" (7, 3.5)
  , label "prune" (13, 3.5)
  , xarrow (0, 5) (2, 5)
  , xarrow (3, 5) (5, 5)
  , xarrow (6, 5) (8, 5)
  , nxarrow (8, 2) (6, 2)
  , nxarrow (5, 2) (3, 2)
```

```
    , nyarrow (8.5, 4.5) (8.5, 2.5)
    , line (9, 2) (11, 2)
    , line (11, 2) (11, 5)
    , nxarrow (11, 5) (9, 5)
    , phantom' (box (-1,-0.5) (14.5, 7))
    ] # centerXY

d'bifur =
  image "bifurcation-grey.png" 16 9 # centerXY
  === strutY 0.5 ===
  (label "2" (0, 0.5) <> label "r" (8, 0.5) <> label "4" (16, 0.5)) # centerXY

d'bifurc =
  image "bifurcation-colour.png" 16 9 # centerXY
  === strutY 0.5 ===
  (label "2" (0, 0.5) <> label "r" (8, 0.5) <> label "4" (16, 0.5)) # centerXY

d'bifurc2 =
  (((label "4" (0.5, 9) <> label "B" (0.5, 4.5) <> label "2" (0.5, 0)) # ↙
      ↳ centerXY)
   ||| strutX 0.5 ||| image "bifurcation-2d.png" 9 9 # centerXY) # centerXY
  === strutY 0.25 ===
  (phantom' (label " " (0.5, 0.5) # centerXY)
   ||| strutX 0.5 |||
   (label "2" (0, 0.5) <> label "A" (4.5, 0.5) <> label "4" (9, 0.5)) # centerXY
  ) # centerXY

dashed = dashing [0.125, 0.125] 0.1875
```

19 examples/cicada-2.sh

```
#!/bin/bash
set -e
s="cicada-2"
w=16
h=9
mkdir -p "${s}"
( cat <<-EOF
        lyapunov-fm/0
        geometry = 1280x720
        tiling   = 1x1+0+0
        delay    = 0.750000
        radius   = 0.262415
        note_x   = 93.291428
        note_y   = 106.493973
        index_x  = 5.687781
        index_y  = -38.408806
        matrix   = [ 1.000000, -1.000000, 0.000000, 0.000000 ; 0.000000, ↙
            ↳ 0.000000, 1.000000, -1.000000 ]
        render(${s}/small.pgm)
        geometry = 256x256
EOF
for y in $(seq $((h - 1)) -1 0)
do
  for x in $(seq 0 $((w - 1)))
  do
    cat <<-EOF
```

```
        tiling    = ${w}x${h}+${x}+${y}
        render(${s}/tile_${y}_${x}.pgm)
EOF
  done
done ) | ../src/lyapunov-fm 0.75 --batch
for y in $(seq 0 $((h - 1)))
do
  pnmcat -lr $(for x in $(seq 0 $((w - 1))) ; do echo "${s}/tile_${y}_${x}.pgm" ↙
      ↳ ; done) > "${s}/row_${y}.pgm"
done
pnmcat -tb $(for y in $(seq $((h - 1)) -1 0) ; do echo "${s}/row_${y}.pgm" ; ↙
    ↳ done) > "${s}/${s}.pgm"
pnmtopng -interlace -compression 9 -force "${s}/${s}.pgm" > "${s}/${s}.png"
convert "${s}/${s}.png" -normalize -geometry "1920x1080" -quality 90 "${s}/${s}.↙
    ↳ jpg"
```

20 examples/cicada-2-zoom.sh

```
#!/bin/bash
set -e
s="cicada--2-zoom"
w=32
h=18
mkdir -p "${s}"
( cat <<-EOF
        lyapunov-fm/0
        delay    = 0.750000
        note_x   = 93.291428
        note_y   = 106.493973
        index_x  = 5.687781
        index_y  = -38.408806
        matrix   = [ 1, -1, 0, 0 ; 0, 0, 1, -1 ]
        geometry = 240x240
EOF
for r in 0.25 0.5 1 2 4 8 16 32 64
do
  mkdir -p "${s}/${r}"
  echo "radius = ${r}"
  for y in $(seq $((h - 1)) -1 0)
  do
    for x in $(seq 0 $((w - 1)))
    do
      cat <<-EOF
        tiling = ${w}x${h}+${x}+${y}
        render(${s}/${r}/tile_${y}_${x}.pgm)
EOF
    done
  done
done ) | ../src/lyapunov-fm 0.75 --batch
for r in 0.25 0.5 1 2 4 8 16 32 64
do
  for y in $(seq 0 $((h - 1)))
  do
    pnmcat -lr $(for x in $(seq 0 $((w - 1))) ; do echo "${s}/${r}/tile_${y}_${x↙
        ↳ }.pgm" ; done) > "${s}/${r}/row_${y}.pgm"
  done
  pnmcat -tb $(for y in $(seq $((h - 1)) -1 0) ; do echo "${s}/${r}/row_${y}.pgm↙
```

```
      ↪ " ; done) > "${s}/${r}.pgm"
  pnmtopng -interlace -compression 9 -force "${s}/${r}.pgm" > "${s}/${r}.png"
  convert "${s}/${r}.png" -normalize -geometry "1920x1080" -quality 90 "${s}/${r↙
      ↪ }.jpg"
done
```

21 examples/cicada.sh

```
#!/bin/bash
set -e
s="cicada"
w=16
h=9
mkdir -p "${s}"
( cat <<-EOF
        lyapunov-fm/0
        geometry = 1280x720
        tiling   = 1x1+0+0
        delay    = 0.750000
        radius   = 0.262415
        note_x   = 93.291428
        note_y   = 106.493973
        index_x  = 5.687781
        index_y  = -38.408806
        matrix   = [ 1.000000, 1.000000, 0.000000, 0.000000 ; 0.000000, ↙
            ↪ 0.000000, 1.000000, 1.000000 ]
        render(${s}/small.pgm)
        geometry = 256x256
EOF
for y in $(seq $((h - 1)) -1 0)
do
  for x in $(seq 0 $((w - 1)))
  do
    cat <<-EOF
        tiling   = ${w}x${h}+${x}+${y}
        render(${s}/tile_${y}_${x}.pgm)
EOF
  done
done ) | ../src/lyapunov-fm 0.75 --batch
for y in $(seq 0 $((h - 1)))
do
  pnmcat -lr $(for x in $(seq 0 $((w - 1))) ; do echo "${s}/tile_${y}_${x}.pgm" ↙
      ↪ ; done) > "${s}/row_${y}.pgm"
done
pnmcat -tb $(for y in $(seq $((h - 1)) -1 0) ; do echo "${s}/row_${y}.pgm" ; ↙
    ↪ done) > "${s}/${s}.pgm"
pnmtopng -interlace -compression 9 -force "${s}/${s}.pgm" > "${s}/${s}.png"
convert "${s}/${s}.png" -normalize -geometry "1920x1080" -quality 90 "${s}/${s}.↙
    ↪ jpg"
```

22 examples/cicada-zoom.sh

```
#!/bin/bash
set -e
s="cicada-zoom"
w=32
```

```
h=18
mkdir -p "${s}"
( cat <<-EOF
        lyapunov-fm/0
        delay    = 0.750000
        note_x   = 93.291428
        note_y   = 106.493973
        index_x  = 5.687781
        index_y  = -38.408806
        matrix   = [ 1, 1, 0, 0 ; 0, 0, 1, 1 ]
        geometry = 240x240
EOF
for r in 0.25 0.5 1 2 4 8 16 32 64
do
  mkdir -p "${s}/${r}"
  echo "radius = ${r}"
  for y in $(seq $((h - 1)) -1 0)
  do
    for x in $(seq 0 $((w - 1)))
    do
      cat <<-EOF
        tiling = ${w}x${h}+${x}+${y}
        render(${s}/${r}/tile_${y}_${x}.pgm)
EOF
    done
  done
done ) | ../src/lyapunov-fm 0.75 --batch
for r in 0.25 0.5 1 2 4 8 16 32 64
do
  for y in $(seq 0 $((h - 1)))
  do
    pnmcat -lr $(for x in $(seq 0 $((w - 1))) ; do echo "${s}/${r}/tile_${y}_${x}.pgm" ; done) > "${s}/${r}/row_${y}.pgm"
  done
  pnmcat -tb $(for y in $(seq $((h - 1)) -1 0) ; do echo "${s}/${r}/row_${y}.pgm" ; done) > "${s}/${r}.pgm"
  pnmtopng -interlace -compression 9 -force "${s}/${r}.pgm" > "${s}/${r}.png"
  convert "${s}/${r}.png" -normalize -geometry "1920x1080" -quality 90 "${s}/${r}.jpg"
done
```

23 examples/crosswired.sh

```
#!/bin/bash
mkdir -p crosswired
w=16
h=12
( cat <<-EOF
        lyapunov-fm/0
        geometry = 1024x768
        tiling   = 1x1+0+0
        delay    = 0.75
        radius   = 0.5
        note_x   = 111.879204
        note_y   = 124.428627
        index_x  = 45.643826
        index_y  = 1.264403
```

```
        matrix   = [ 1, 1, 0, 0 ; 0, 0, 1, 1 ]
        render(crosswired/crosswired_preview.pgm)
        geometry = 600x600
EOF
for y in $(seq 0 $((h - 1)))
do
  for x in $(seq 0 $((w - 1)))
  do
    cat <<-EOF
        tiling   = ${w}x${h}+${x}+${y}
        render(crosswired/tile_${y}_${x}.pgm)
EOF
  done
done ) | ../src/lyapunov-fm 0.75 --batch
for y in $(seq 0 $((h - 1)))
do
  pnmcat -lr $(for x in $(seq 0 $((w - 1))) ; do echo "crosswired/tile_${y}_${x}.pgm" ; done) > "crosswired/row_${y}.pgm"
done
pnmcat -tb $(for y in $(seq $((h - 1)) -1 0) ; do echo "crosswired/row_${y}.pgm" ; done) > "crosswired/crosswired.pgm"
```

24 examples/delay.sh

```
#!/bin/bash
set -e
s="delay"
mkdir -p "${s}"
for d in $(seq 0 27)
do
  cat <<-EOF | ../src/lyapunov-fm ${d} --batch
        lyapunov-fm/0
        geometry = 1280x720
        tiling   = 1x1+0+0
        delay    = ${d}
        radius   = 48
        note_x   = 96
        note_y   = 96
        index_x  = 0
        index_y  = 0
        matrix   = [ 1, -1, 0, 0 ; 0, 0, 1, -1 ]
        render(${s}/${d}.pgm)
EOF
  pnmtopng -interlace -compression 9 -force "${s}/${d}.pgm" > "${s}/${d}.png"
done
```

25 fragmentarium/ab.frag

```
#include "Progressive2D.frag"

vec3 color(vec2 ab) {
        float rB = ab.x + 3.0;
        float rA = ab.y + 3.0;
        if (0.0 < rA && rA < 4.0 && 0.0 < rB && rB < 4.0) {
                float l = 0.0;
                float n = 0.0;
```

```
                float z = 0.5;
                float dz;
                for (int i = 0; i < 128; ++i) {
                        z = rA * z * (1.0 - z);
                        z = rB * z * (1.0 - z);
                }
                for (int i = 0; i < 1024; ++i) {
                        z = rA * z * (1.0 - z);
                        dz = abs(rA * (1.0 - 2.0 * z));
                        if (dz > 0.0) {
                                dz = log(dz);
                                if (-1000.0 < dz && dz < 1000.0) {
                                        l += dz;
                                        n += 1.0;
                                }
                        }
                        z = rB * z * (1.0 - z);
                        dz = abs(rB * (1.0 - 2.0 * z));
                        if (dz > 0.0) {
                                dz = log(dz);
                                if (-1000.0 < dz && dz < 1000.0) {
                                        l += dz;
                                        n += 1.0;
                                }
                        }
                }
                if (n > 0.0) {
                        l /= n;
                        if (l <= 0.01) {
                                return vec3(1.0 + tanh(l / 4.0), 1.0 + tanh(l / ↙
                                    ↳ 3.0), 0.0);
                        } else {
                                return vec3(0.0, 0.0, tanh(l * 3.0));
                        }
                } else {
                        return vec3(1.0, 1.0, 0.0);
                }
        } else {
                return vec3(0.0);
        }
}
```

26 .gitignore

```
-
src/*.glsl.c
src/lyapunov-fm
src/overlay
doc/lyapunov-fm.aux
doc/lyapunov-fm.bbl
doc/lyapunov-fm.blg
doc/lyapunov-fm.log
doc/lyapunov-fm.pdf
doc/videos
*.pgm
*.ppm
*.png
```

```
*.jpg
*.mkv
*.ogv
```

27 README

```
lyapunov-fm -- Lyapunov space of coupled FM oscillators
Copyright (C) 2013 Claude Heiland-Allen <claude@mathr.co.uk>
License: GPLv3+ http://www.gnu.org/licenses/gpl.html


Usage
-----

make -C src && ./src/lyapunov-fm

mouse left/scrollup: zoom in
mouse right/scrolldown: zoom out
mouse middle: recenter view
TAB: toggle viewing plane
F11: toggle full screen


Documentation
-------------

make -C doc && open doc/lyapunov-fm.pdf

Note: the figures for the paper are not included in this repository, see:
http://mathr.co.uk/lyapunov-fm/
```

28 src/audio.c

```
#include "audio.h"

static inline float mtoi(float m, float sr) {
  return 440.0f * powf(2.0f, (m - 69.0f) / 12.0f) / sr;
}

static inline float mix(float x, float y, float f) {
  return x * f + (1 - f) * y;
}
static inline float wmix(float x, float y, float f) {
  if (y < x) { return mix(x, y + 1, f); }
  else       { return mix(x, y, f); }
}

int audio_process(jack_nframes_t nframes, void *arg) {
  struct audio *s = (struct audio *) arg;
  jack_default_audio_sample_t *out[4];
  out[0] = (jack_default_audio_sample_t *) jack_port_get_buffer(s->port[0], ↙
      ↳ nframes);
  out[1] = (jack_default_audio_sample_t *) jack_port_get_buffer(s->port[1], ↙
      ↳ nframes);
  out[2] = (jack_default_audio_sample_t *) jack_port_get_buffer(s->port[2], ↙
      ↳ nframes);
```

```
  out[3] = (jack_default_audio_sample_t *) jack_port_get_buffer(s->port[3], ↙
      ↳ nframes);
  float *phase = s->state, df = s->df, sr = 48000.0f, m[2], p[4], Q[2];
  int j = s->j, d = s->d, dd = 2 * (s->d + 1);
  m[0] = s->mouse[0];
  m[1] = s->mouse[1];
  p[0] = s->param[0];
  p[1] = s->param[1];
  p[2] = s->param[2];
  p[3] = s->param[3];
  for (jack_nframes_t i = 0; i < nframes; ++i) {
#define P(w,t) phase[(2 * (t) + w + dd)%dd]
    Q[0] = fmodf(P(0,j) + mtoi(p[0] + p[2] * (out[0][i] = cosf(6.283185307179586↙
        ↳ f * wmix(P(1,j-d), P(1,j-d+1), df))), sr), 1.0f);
    Q[1] = fmodf(P(1,j) + mtoi(p[1] + p[3] * (out[1][i] = cosf(6.283185307179586↙
        ↳ f * wmix(P(0,j-d), P(0,j-d+1), df))), sr), 1.0f);
#define NOISE (0.00001f * (rand() / (float)RAND_MAX - 0.5f))
    P(0,j+1) = Q[0] + NOISE;
    P(1,j+1) = Q[1] + NOISE;
#undef NOISE
#undef P
    out[2][i] = m[0];
    out[3][i] = m[1];
    j = (j + 1) % (d + 1);
  }
  s->j = j;
  return 0;
}

void audio_do(struct audio *s, glm::vec2 &m, glm::vec4 &p) {
  s->mouse[0] = m[0];
  s->mouse[1] = m[1];
  s->param[0] = p[0];
  s->param[1] = p[1];
  s->param[2] = p[2];
  s->param[3] = p[3];
}

void audio_begin(struct audio *s, int d, float df) {
  s->mouse[0] = 0;
  s->mouse[1] = 0;
  s->param[0] = 48;
  s->param[1] = 48;
  s->param[2] = 0;
  s->param[3] = 0;
  s->d = d;
  s->df = df;
  s->j = 0;
  s->state = (float *) calloc(2 * (d + 1), sizeof(float));
  if ((s->client = jack_client_open("lyapunov-fm", JackNoStartServer, 0))) {
    jack_set_process_callback(s->client, audio_process, s);
    s->port[0] = jack_port_register(s->client, "output_1", ↙
        ↳ JACK_DEFAULT_AUDIO_TYPE, JackPortIsOutput, 0);
    s->port[1] = jack_port_register(s->client, "output_2", ↙
        ↳ JACK_DEFAULT_AUDIO_TYPE, JackPortIsOutput, 0);
    s->port[2] = jack_port_register(s->client, "output_3", ↙
        ↳ JACK_DEFAULT_AUDIO_TYPE, JackPortIsOutput, 0);
```

```
    s->port[3] = jack_port_register(s->client, "output_4", ↙
        ↳ JACK_DEFAULT_AUDIO_TYPE, JackPortIsOutput, 0);
    if (jack_activate(s->client)) {
      fprintf (stderr, "cannot activate JACK client\n");
    } else {
      const char **ports;
      if ((ports = jack_get_ports(s->client, NULL, NULL, JackPortIsPhysical | ↙
          ↳ JackPortIsInput))) {
        for (int i = 0; i < 2; ++ i) {
          if (! ports[i]) {
            break;
          }
          if (jack_connect(s->client, jack_port_name(s->port[i]), ports[i])) {
            fprintf(stderr, "cannot connect JACK output port\n");
          }
        }
        free(ports);
      }
    }
  }
}

void audio_end(struct audio *s) {
  if (s->client) {
    jack_client_close(s->client);
  }
}
```

29 src/audio.h

```
#ifndef AUDIO_H
#define AUDIO_H 1

#include <jack/jack.h>

struct audio {
  jack_client_t *client;
  jack_port_t *port[4];
  float mouse[2];
  float param[4];
  int d;
  float df;
  int j;
  float *state;
};

void audio_begin(struct audio *s, int d, float df);
void audio_end(struct audio *s);
void audio_do(struct audio *s, glm::vec2 &m, glm::vec4 &p);

#endif
```

30 src/batch.c

```
void batch_begin(struct batch *b) {
  b->version = -1;
```

```
  b->size = 0;
  view_init(&b->v, 1024, 576, 120, 120, 0, 0, 72, 0);
  b->size_specified = 0;
}

void batch_done(struct render *r, void *v) {
  (void) r;
  struct batch *b = (struct batch *) v;
  S.record.screenshot = 1;
  record_do(&S.record, &b->v);
  batch_do(b, &S.record);
}

void batch_do(struct batch *b, struct record *rec) {
  int len;
#define L if (-1 == getline(&b->line, &b->size, stdin)) { exit(0); } len = strlen(b->line); if (b->line[len - 1] == '\n') { b->line[len - 1] = 0; len--; } fprintf(stderr, "%s\n", b->line);
  char *rest = 0;
#define S0(str) if (0 == strncmp(str, b->line, strlen(str))) { rest = b->line + strlen(str);
#define S(str) } else S0(str)
#define F(fld) sscanf(rest, " = %f", &b->v.fld);
  L
  S0("lyapunov-fm") if (0 == strcmp("/0", rest)) { b->version = 0; L } else { fprintf(stderr, "bad version: %s\n", b->line); exit(1); }
  } while (1) {
    S0("geometry") sscanf(rest, " = %dx%d", &b->v.width, &b->v.height); b->size_specified = 1;
    S("tiling")    sscanf(rest, " = %dx%d+%d+%d", &b->v.tilesx, &b->v.tilesy, &b->v.tilex, &b->v.tiley);
    S("delay")     F(delay)
    S("radius")    F(radius)
    S("note_x")    F(center[0])
    S("note_y")    F(center[1])
    S("index_x")   F(center[2])
    S("index_y")   F(center[3])
    S("matrix")    sscanf(rest, " = [ %f, %f, %f, %f ; %f, %f, %f, %f ]"
      , &b->v.matrix[0][0], &b->v.matrix[0][1], &b->v.matrix[0][2], &b->v.matrix[0][3]
      , &b->v.matrix[1][0], &b->v.matrix[1][1], &b->v.matrix[1][2], &b->v.matrix[1][3]);
    S("render")    if (rest[0] == '(' && rest[strlen(rest) - 1] == ')') { rest[strlen(rest) - 1] = 0; if (rec->filename) { free(rec->filename); } rec->filename = strdup(rest + 1); break; }
    } else { fprintf(stderr, "warning: ignoring: %s\n", b->line);
    }
    L
  }
#undef S0
#undef S
#undef F
#undef L
  render_ondone(&S.render, batch_done, b);
  render_do(&S.render, &b->v);
}
```

```
void batch_reshape(struct batch *b, int w, int h) {
  view_reshape(&b->v, w, h);
}
```

31 src/batch.h

```
#ifndef BATCH_H
#define BATCH_H 1

struct batch {
  int version;
  char *line;
  size_t size;
  struct view v;
  int size_specified;
};

void batch_begin(struct batch *b);
void batch_do(struct batch *b, struct record *rec);

#endif
```

32 src/colour.c

```
#include "colour_vert.glsl.c"
#include "colour_frag.glsl.c"

struct colour {
  GLuint program;
  GLint mvp;
  GLint tex;
  GLint q0;
  GLuint vao;
  GLuint vbo;
};

void colour_begin(struct colour *s) {
  s->program = 0;
  s->mvp = -1;
  s->tex = -1;
  s->q0 = -1;
  s->vao = 0;
  s->vbo = 0;
  s->program = compile_program("colour", colour_vert, 0, colour_frag);
  s->mvp = glGetUniformLocation(s->program, "mvp");D;
  s->tex = glGetUniformLocation(s->program, "tex");D;
  s->q0 = glGetAttribLocation(s->program, "q0");D;
  glGenVertexArrays(1, &s->vao);D;
  glGenBuffers(1, &s->vbo);D;
}

void colour_end(struct colour *s) {
  glDeleteProgram(s->program);D;
  s->program = 0;
  glDeleteVertexArrays(1, &s->vao);D;
  s->vao = 0;
```

```
  glDeleteBuffers(1, &s->vbo);D;
  s->vbo = 0;
}

int cmp_float(const void *x, const void *y) {
  const float *a = (const float *) x;
  const float *b = (const float *) y;
  if (*a < *b) return -1;
  if (*a > *b) return  1;
  return 0;
}

void colour_do(struct colour *s, GLuint tex, int tex_size, struct view *v) {
  glBindTexture(GL_TEXTURE_2D, tex);D;
  glUseProgram(s->program);D;
  glm::mat4 mvp = glm::ortho(0.0f, 1.0f, 0.0f, 1.0f);
  glUniformMatrix4fv(s->mvp, 1, GL_FALSE, &mvp[0][0]);D;
  glUniform1i(s->tex, 0);D;
  GLfloat x = v->width  * 1.0 / tex_size;
  GLfloat y = v->height * 1.0 / tex_size;
  GLfloat quad[] = {
    0, 0,  0, 0,
    0, 1,  0, y,
    1, 0,  x, 0,
    1, 1,  x, y
  };
  glBindVertexArray(s->vao);D;
  glBindBuffer(GL_ARRAY_BUFFER, s->vbo);D;
  glBufferData(GL_ARRAY_BUFFER, 16 * sizeof(GLfloat), &quad, GL_STATIC_DRAW);D;
  glEnableVertexAttribArray(s->q0);D;
  glVertexAttribPointer(s->q0, 4, GL_FLOAT, GL_FALSE, 0, 0);D;
  glDrawArrays(GL_TRIANGLE_STRIP, 0, 4);D;
  glDisableVertexAttribArray(s->q0);D;
  glBindBuffer(GL_ARRAY_BUFFER, 0);D;
  glBindVertexArray(0);D;
  glUseProgram(0);D;
  glBindTexture(GL_TEXTURE_2D, 0);D;
}
```

33 src/colour_frag.glsl

```
uniform sampler2D tex;
smooth in vec2 t;
out layout(location = 0, index = 0) vec4 c;
void main() {
  vec3 g = texture(tex, t).xyz;
  float mean = 0.0;
  if (g.x > 0.0) { mean = g.y / g.x; }
  float umean = (g.y + 10.0) / (g.x + 1.0);
  float lmean = (g.y - 10.0) / (g.x + 1.0);
  float delta = abs(tanh(8.0 * umean) - tanh(8.0 * lmean));
  if (delta < 0.5 / 256.0) {
    c = vec4(vec3(0.5 + 0.5 * tanh(8.0 * mean)), 1.0);
  } else {
    c = vec4(vec3(0.5 + 0.5 * tanh(8.0 * mean)), 1.0) * vec4(1.0, 0.7, 0.7, 1.0)↙
        ↳ ;
  }
```

```
/*
  if (l <= -10.0) {
    c = vec4(0.0, 0.0, 0.0, 1.0);
  } else {
    c = 0.5 * mix(vec4(1.0 - tanh(l * 4.0), 1.0 - tanh(l * 3.0), 0.0, 2.0),
            vec4(0.0, 0.0, tanh(l * 3.0) + 1.0, 2.0), 0.5 + 0.5 * tanh(l));
  }
*/
}
```

34 src/colour_vert.glsl

```
uniform mat4 mvp;
in vec4 q0;
smooth out vec2 t;
void main() {
  t = q0.zw;
  gl_Position = mvp * vec4(q0.xy, 0.0, 1.0);
}
```

35 src/dvariable.c

```
void dvariable_begin(struct dvariable *s, int d) {
  s->d = d;
  s->packed = (d >> 1) + 1;
  s->nvaryings = s->packed + 2;
  s->attributes = (GLchar **) calloc(s->nvaryings, sizeof(GLchar *));
  s->varyings = (GLchar **) calloc(s->nvaryings, sizeof(GLchar *));
  for (int i = 0; i < s->packed; ++i) {
    s->attributes[i] = (GLchar *) calloc(1, 8);
    snprintf(s->attributes[i], 7, "p%d_i", i);
    s->varyings[i] = (GLchar *) calloc(1, 8);
    snprintf(s->varyings[i], 7, "p%d_o", i);
  }
  s->attributes[s->packed    ] = strdup("g_i");
  s->attributes[s->packed + 1] = strdup("q_i");
  s->varyings[s->packed    ] = strdup("g_o");
  s->varyings[s->packed + 1] = strdup("q_o");
  s->stride = (4 * (s->packed + 1) + 2) * sizeof(GLfloat);
  s->gptr = ((GLbyte *)0)+4*s->packed*sizeof(GLfloat);
  s->qptr = ((GLbyte *)0)+4*(s->packed+1)*sizeof(GLfloat);
}

void dvariable_setpointers(struct dvariable *s, GLuint program) {
  for (int i = 0; i < s->packed; ++i) {
    GLint p0 = glGetAttribLocation(program, s->attributes[i]);D;
    glEnableVertexAttribArray(p0);D;
    glVertexAttribPointer(p0, 4, GL_FLOAT, GL_FALSE, s->stride, ((GLbyte *)0)+ i↙
        ↳ *(4*sizeof(GLfloat)));D;
  }
  GLint g0 = glGetAttribLocation(program, "g_i");D;
  glEnableVertexAttribArray(g0);D;
  glVertexAttribPointer(g0, 4, GL_FLOAT, GL_FALSE, s->stride, s->gptr);D;
  GLint q0 = glGetAttribLocation(program, "q_i");D;
  glEnableVertexAttribArray(q0);D;
  glVertexAttribPointer(q0, 2, GL_FLOAT, GL_FALSE, s->stride, s->qptr);D;
```

```
}
```

36 src/dvariable.h

```
#ifndef DVARIABLE_H
#define DVARIABLE_H 1

struct dvariable {
  int d;
  int packed;
  int nvaryings;
  GLchar **attributes; // p%d_i, g_i, q_i
  GLchar **varyings;   // p%d_o, g_o, q_o
  int stride;
  void *gptr;
  void *qptr;
};

void dvariable_begin(struct dvariable *s, int d);
void dvariable_setpointers(struct dvariable *s, GLuint program);

#endif
```

37 src/fillc.c

```
#include "fillc_vert.glsl.c"
#include "fillc_frag.glsl.c"

struct fillc {
  GLuint program;
  GLint  mvp;
  GLint  p;
  GLuint vao;
  GLuint vbo;
};

void fillc_begin(struct fillc *s) {
  s->program = 0;
  s->mvp = -1;
  s->p = -1;
  s->vbo = 0;
  s->vao = 0;
  s->program = compile_program("fillc", fillc_vert, 0, fillc_frag);
  s->mvp = glGetUniformLocation(s->program, "mvp");D;
  s->p = glGetAttribLocation(s->program, "p");D;
  glGenVertexArrays(1, &s->vao);D;
  glGenBuffers(1, &s->vbo);D;
}

void fillc_end(struct fillc *s) {
  glDeleteProgram(s->program);D;
  s->program = 0;
  glDeleteVertexArrays(1, &s->vao);D;
  s->vao = 0;
  glDeleteBuffers(1, &s->vbo);D;
  s->vbo = 0;
```

```
}

void fillc_do(struct fillc *s, GLuint vbo1, GLuint fbo, GLuint tex, struct view ↙
    ↳ *v) {
  glBindFramebuffer(GL_FRAMEBUFFER, fbo);D;
  glFramebufferTexture(GL_FRAMEBUFFER, GL_COLOR_ATTACHMENT0, tex, 0);D;
  GLenum buffers[1] = { GL_COLOR_ATTACHMENT0 };
  glDrawBuffers(1, buffers);D;
  glViewport(0, 0, v->width, v->height);D;
  glUseProgram(s->program);D;
  float aspect = v->width * 1.0f / v->height;
  float tw = 2.0f * aspect / v->tilesy;
  float th = 2.0f          / v->tilesy;
  float tx = (v->tilex - v->tilesx / 2.0) * tw;
  float ty = (v->tiley - v->tilesy / 2.0) * th;
  glm::mat4 mvp = glm::ortho(tx, tx + tw, ty, ty + th);
  glUniformMatrix4fv(s->mvp, 1, GL_FALSE, &mvp[0][0]);D;
  GLfloat quad[] = {
    tx,      ty,
    tx,      ty + th,
    tx + tw, ty,
    tx + tw, ty + th
  };
  glBindVertexArray(s->vao);D;
  glBindBuffer(GL_ARRAY_BUFFER, s->vbo);D;
  glBufferData(GL_ARRAY_BUFFER, 8 * sizeof(GLfloat), &quad, GL_STATIC_DRAW);D;
  glVertexAttribPointer(s->p, 2, GL_FLOAT, GL_FALSE, 0, 0);D;
  glEnableVertexAttribArray(s->p);D;
  glDrawArrays(GL_TRIANGLE_STRIP, 0, 4);D;
  glDisableVertexAttribArray(s->p);D;
  glBindBuffer(GL_ARRAY_BUFFER, 0);D;
  glBindVertexArray(0);D;
  glUseProgram(0);D;
  // Copy texture to vertex buffer.
  glBindBuffer(GL_PIXEL_PACK_BUFFER, vbo1);D;
  glReadPixels(0, 0, v->width, v->height, GL_RG, GL_FLOAT, 0);D;
  glBindBuffer(GL_PIXEL_PACK_BUFFER, 0);D;
  glClearColor(0,0,0,0);D;
  glClear(GL_COLOR_BUFFER_BIT);D;
  glBindFramebuffer(GL_FRAMEBUFFER, 0);D;
}
```

38 src/fillc_frag.glsl

```
smooth in vec2 t;
out layout(location = 0, index = 0) vec4 o;
void main() {
  o = vec4(t, 0.0, 0.0);
}
```

39 src/fillc_vert.glsl

```
uniform mat4 mvp;
in  vec2 p;
smooth out vec2 t;
void main() {
```

```
  gl_Position = mvp * vec4(p, 0.0, 1.0);
  t = p;
}
```

40 src/init.c

```
struct init {
  GLuint program;
  GLint c;
  GLuint vao;
};

void init_begin(struct init *s, struct dvariable *d) {
  s->program = 0;
  s->c = -1;
  s->vao = 0;
  int len = 65536;
  char *src = (char *) calloc(1, len);
  int slen = 0;
#define L0(str) slen += snprintf(src + slen, len - slen - 1, str)
#define L1(fmt,arg) slen += snprintf(src + slen, len - slen - 1, fmt, arg)
  L0("in  vec2 c;\n");
  for (int i = 0; i < d->packed; ++i) {
    L1("flat out vec4 %s;\n", d->varyings[i]);
  }
  L0("flat out vec4 g_o;\n");
  L0("flat out vec2 q_o;\n");
  L0("void main() {\n");
  for (int i = 0; i < d->packed; ++i) {
    L1("  %s = vec4(0.0);\n", d->varyings[i]);
  }
  L0("  g_o = vec4(0.0);\n");
  L0("  q_o = c;\n");
  L0("}\n");
#undef L0
#undef L1
  s->program = compile_program("init", src, 0, 0, d->nvaryings, d->varyings);
  s->c = glGetAttribLocation(s->program, "c");D;
  glGenVertexArrays(1, &s->vao);D;
}

void init_end(struct init *s) {
  glDeleteProgram(s->program);D;
  s->program = 0;
  glDeleteVertexArrays(1, &s->vao);D;
  s->vao = 0;
}

void init_do(struct init *s, GLuint vbo0, GLuint vbo1, struct view *v) {
  glEnable(GL_RASTERIZER_DISCARD);D;
  glBindVertexArray(s->vao);D;
  glBindBuffer(GL_ARRAY_BUFFER, vbo0);D;
  glUseProgram(s->program);D;
  glVertexAttribPointer(s->c, 2, GL_FLOAT, GL_FALSE, 0, 0);D;
  glEnableVertexAttribArray(s->c);D;
  glBindBufferBase(GL_TRANSFORM_FEEDBACK_BUFFER, 0, vbo1);D;
  glBeginTransformFeedback(GL_POINTS);D;
```

```
  glDrawArrays(GL_POINTS, 0, v->width * v->height);D;
  glEndTransformFeedback();D;
  glBindBufferBase(GL_TRANSFORM_FEEDBACK_BUFFER, 0, 0);D;
  glUseProgram(0);D;
  glBindBuffer(GL_ARRAY_BUFFER, 0);D;
  glBindVertexArray(0);D;
  glDisable(GL_RASTERIZER_DISCARD);D;
}
```

41 src/interact.c

```
static struct {
  int which;
  struct view view;
} I;

void interact_mouse(int button, int state, int x, int y) {
  if (state == GLUT_DOWN) {
    glm::vec4 v;
    view_coord(&v, &I.view, x, y);
    if (button == GLUT_MIDDLE_BUTTON) { view_center(&I.view, v);
    } else if (button == GLUT_LEFT_BUTTON  || button == 3) { view_zoom(&I.view, ↙
        ↳ v, 1);
    } else if (button == GLUT_RIGHT_BUTTON || button == 4) { view_zoom(&I.view, ↙
        ↳ v, -1);
    }
    render_do(&S.render, &I.view);
  }
}

void interact_motion(int x, int y) {
  glm::vec4 v;
  view_coord(&v, &I.view, x, y);
  glm::vec2 m;
  m[0] = x / (float) I.view.width;
  m[1] = y / (float) I.view.height;
  audio_do(&S.audio, m, v);
}

void interact_automate(int x) {
  if (x < I.view.width) {
    interact_motion(x, I.view.height / 2);
    glutTimerFunc(40, interact_automate, x + 1);
  } else {
    interact_motion(0, I.view.height / 2);
    jack_transport_stop(S.audio.client);
  }
}

void interact_keyboard(unsigned char key, int x, int y) {
  (void) x;
  (void) y;
  if (key == 27 || key == 'q') {
    exit(0);
  }
  if (S.mode != 0) return;
  if (key == '\t') {
```

```
45      I.which = -I.which;
        view_plane(&I.view, I.which);
        render_do(&S.render, &I.view);
    /*
      } else if (key == 'a') {
50      S.record.screenshot = true;
        glutPostRedisplay();
        jack_transport_start(S.audio.client);
        glutTimerFunc(40, interact_automate, 0);
      } else if (key == 'l') {
55      S.animate.active = 1;
        S.render.view.which = 2;
        glutPostRedisplay();
    */
      } else if (key == 's') {
60      S.record.screenshot = true;
        glutPostRedisplay();
    /*
      } else if (key == 'r') {
        S.record.video = !S.record.video;
65      glutPostRedisplay();
      } else if (key == '@') {
        S.render.idle.f = 0;
        S.rendering = true;
    */
70    }
    }

    void interact_special(int key, int x, int y) {
      (void) x;
75    (void) y;
      if (key == GLUT_KEY_F11)                { glutFullScreenToggle(); return;
      } else if (key == GLUT_KEY_HOME)       { view_reset(&I.view, 96, 96, 0, 0, 48);↙
          ↪ view_plane(&I.view, I.which = 1);
      } else if (key == GLUT_KEY_PAGE_UP)    { view_zoom(&I.view,  0.1);
      } else if (key == GLUT_KEY_PAGE_DOWN) { view_zoom(&I.view, -0.1);
80    } else                                  { return; }
      render_do(&S.render, &I.view);
    }

    void interact_reshape(int w, int h) {
85    view_reshape(&I.view, w, h);
    }

    void interact_begin() {
      I.which = 1;
90    view_init(&I.view, 1024, 576, 96, 96, 0, 0, 48, 0);
      glutMouseFunc(interact_mouse);
      glutPassiveMotionFunc(interact_motion);
      glutKeyboardFunc(interact_keyboard);
      glutSpecialFunc(interact_special);
95  }
```

42 src/main.cc

```
#define _POSIX_C_SOURCE 199309L
```

```
#include <GL/glew.h>
#include <GL/freeglut.h>
#include <glm/glm.hpp>
#include <glm/gtc/matrix_transform.hpp>
#include <string.h>
#include <stdio.h>
#include <time.h>

#define D do{ int e = glGetError(); if (e != 0) { fprintf(stderr, "OpenGL error ↙
    ↳ %d in %s() (line %d)\n", e, __FUNCTION__, __LINE__); } }while(0)

#include "utility.c"
#include "shader.c"
#include "record.c"

#include "dvariable.h"
#include "dvariable.c"
#include "fillc.c"
#include "init.c"
#include "step.c"
#include "prune.c"
#include "plot.c"
#include "colour.c"

#include "state.c"
void idlecb();
#include "interact.c"
#include "render.c"
#include "view.c"
#include "audio.c"
#include "batch.c"

void reshapecb(int new_width, int new_height) {
  glViewport(0, 0, new_width, new_height);

  if (S.record.buffer) {
    free(S.record.buffer);
    S.record.buffer = 0;
  }

  render_reshape(&S.render, new_width, new_height);
  if (S.mode == 0) {
    interact_reshape(new_width, new_height);
    render_do(&S.render, &I.view);
  } else {
    batch_reshape(&S.batch, new_width, new_height);
    render_do(&S.render, &S.batch.v);
  }

  glutIdleFunc(idlecb);
}

void begincb(int d, float df) {
  render_begin(&S.render, d, df);
  if (S.mode == 0) {
    audio_begin(&S.audio, d, df);
    interact_begin();
```

```
  } else {
    batch_begin(&S.batch);
  }
}

void endcb() {
  render_end(&S.render);
  if (S.mode == 0) {
    audio_end(&S.audio);
  }
}

void displaycb() {
  render_display(&S.render);
}

void idlecb() {
  render_idle(&S.render);
}

int main(int argc, char **argv) {
  srand(time(NULL));
  float delay = 1;
  if (argc > 1) {
    delay = atof(argv[1]);
  }
  int d = ceilf(delay);
  float df = d - delay;
  int mode = 0;
  if (argc > 2) {
    mode = 1;
  }
  state_init(mode, delay);
  glutInitWindowSize(512, 288);
  glutInit(&argc, argv);
  glutInitDisplayMode(GLUT_RGBA | GLUT_DOUBLE);
  glutCreateWindow(argv[0]);
  glewExperimental = GL_TRUE;
  glewInit();

  GLint n;
  glGetIntegerv(GL_MAX_VERTEX_ATTRIBS, &n);
  fprintf(stderr, "GL_MAX_VERTEX_ATTRIBS      = %d\n", n);
  glGetIntegerv(GL_MAX_VARYING_COMPONENTS, &n);
  fprintf(stderr, "GL_MAX_VARYING_COMPONENTS = %d\n", n);
  glGetIntegerv(GL_MAX_VARYING_VECTORS, &n);
  fprintf(stderr, "GL_MAX_VARYING_VECTORS     = %d\n", n);
  glGetIntegerv(GL_MAX_VARYING_FLOATS, &n);
  fprintf(stderr, "GL_MAX_VARYING_FLOATS      = %d\n", n);

  begincb(d, df);
  atexit(endcb);
  glutDisplayFunc(displaycb);
  glutReshapeFunc(reshapecb);

  if (mode != 0) {
    batch_do(&S.batch, &S.record);
```

```
  }
  glutMainLoop();
  return 0;
}
```

43 src/Makefile

```
SOURCES = \
        audio.c \
        audio.h \
        batch.c \
        batch.h \
        dvariable.h \
        dvariable.c \
        fillc.c \
        init.c \
        step.c \
        prune.c \
        plot.c \
        colour.c \
        interact.c \
        main.cc \
        record.c \
        render.c \
        render.h \
        shader.c \
        state.c \
        utility.c \
        view.c \
        view.h

SHADERS = \
        preamble.glsl.c \
        fillc_frag.glsl.c \
        fillc_vert.glsl.c \
        plot_vert.glsl.c \
        plot_frag.glsl.c \
        colour_vert.glsl.c \
        colour_frag.glsl.c

all: lyapunov-fm overlay

clean:
        -rm -f lyapunov-fm overlay $(SHADERS)

.SUFFIXES:
.PHONY: all clean

lyapunov-fm: $(SOURCES) $(SHADERS)
        g++ -ggdb -Wall -pedantic -Wextra -O3 -march=native -o lyapunov-fm main.↙
            ↳ cc -lGLEW -lglut -lGL -lrt -ljack

%.glsl.c: %.glsl s2c.sh
        bash s2c.sh $* < $< > $@

overlay: overlay.c
        gcc -std=c99 -ggdb -Wall -pedantic -Wextra -O3 -march=native -o overlay ↙
```

↳ overlay.c

44 src/overlay.c

```c
#include <stdio.h>
#include <stdlib.h>

static inline int min(int x, int y) { return x < y ? x : y; }
static inline int max(int x, int y) { return x > y ? x : y; }

int main(int argc, char **argv) {
  (void) argc;
  (void) argv;
  // read input header
  int w, h;
  if (2 != scanf("P5\n#%*[^\n]\n%d %d 255\n", &w, &h)) { return 1; }
  // allocate buffers
  unsigned char *pgm = calloc(1, w * h);
  if (! pgm) { return 1; }
  unsigned char *ppm = calloc(1, w * h * 3);
  if (! ppm) { return 1; }
  // read input image
  if (1 != fread(pgm, w * h, 1, stdin)) { return 1; }
  // copy image
  for (int i = 0, j = 0; i < w * h; ++i) {
    unsigned char g = pgm[i];
    ppm[j++] = g; ppm[j++] = g; ppm[j++] = g;
  }
  for (int f = 0; f < w; ++f) {
    // draw crosshair
    int x, y;
    y = h / 2;
    for (x = max(0, f - 8); x <= min(f + 8, w - 1); ++x) {
      if (x == f) { continue; }
      int j = 3 * (w * y + x);
      ppm[j++] = 255; ppm[j++] = 0; ppm[j++] = 0;
    }
    x = f;
    for (y = max(0, h/2 - 8); y <= min(h/2 + 8, h - 1); ++y) {
      if (y == h/2) { continue; }
      int j = 3 * (w * y + x);
      ppm[j++] = 255; ppm[j++] = 0; ppm[j++] = 0;
    }
    // write image
    fprintf(stdout, "P6\n%d %d\n255\n", w, h);
    fwrite(ppm, w * h * 3, 1, stdout);
    fflush(stdout);
    // erase crosshair
    y = h/2;
    for (x = max(0, f - 8); x <= min(f + 8, w - 1); ++x) {
      if (x == f) { continue; }
      int i = w * y + x;
      int j = 3 * i;
      unsigned char g = pgm[i];
      ppm[j++] = g; ppm[j++] = g; ppm[j++] = g;
    }
    x = f;
```

```
      for (y = max(0, h/2 - 8); y <= min(h/2 + 8, h - 1); ++y) {
        if (y == h/2) { continue; }
        int i = w * y + x;
        int j = 3 * i;
        unsigned char g = pgm[i];
        ppm[j++] = g; ppm[j++] = g; ppm[j++] = g;
      }
    }
    return 0;
}
```

45 src/plot.c

```
#include "plot_vert.glsl.c"
#include "plot_frag.glsl.c"

struct plot {
  GLuint program;
  GLint mvp;
  GLint g0;
  GLint q0;
  GLuint vao;
};

void plot_begin(struct plot *s, GLuint vbo, struct dvariable *d) {
  s->program = 0;
  s->mvp = -1;
  s->g0 = -1;
  s->q0 = -1;
  s->vao = 0;
  s->program = compile_program("plot", plot_vert, 0, plot_frag);
  s->mvp = glGetUniformLocation(s->program, "mvp");D;
  s->g0 = glGetAttribLocation(s->program, "g0");D;
  s->q0 = glGetAttribLocation(s->program, "q0");D;
  glGenVertexArrays(1, &s->vao);D;
  glBindVertexArray(s->vao);D;
  glBindBuffer(GL_ARRAY_BUFFER, vbo);D;
  glEnableVertexAttribArray(s->g0);D;
  glEnableVertexAttribArray(s->q0);D;
  glVertexAttribPointer(s->g0, 4, GL_FLOAT, GL_FALSE, d->stride, d->gptr);D;
  glVertexAttribPointer(s->q0, 2, GL_FLOAT, GL_FALSE, d->stride, d->qptr);D;
  glBindBuffer(GL_ARRAY_BUFFER, 0);D;
  glBindVertexArray(0);D;
}

void plot_end(struct plot *s) {
  glDeleteProgram(s->program);D;
  s->program = 0;
  glDeleteVertexArrays(1, &s->vao);D;
  s->vao = 0;
}

void plot_start(struct plot *s, GLuint fbo, GLuint tex, struct view *v) {
  glUseProgram(s->program);D;
  float aspect = v->width * 1.0f / v->height;
  float tw = 2.0f * aspect / v->tilesy;
  float th = 2.0f          / v->tilesy;
```

```
45    float tx = (v->tilex - v->tilesx / 2.0) * tw;
      float ty = (v->tiley - v->tilesy / 2.0) * th;
      glm::mat4 mvp = glm::ortho(tx, tx + tw, ty, ty + th);
      glUniformMatrix4fv(s->mvp, 1, GL_FALSE, &mvp[0][0]);D;
      glUseProgram(0);
50    glBindFramebuffer(GL_FRAMEBUFFER, fbo);D;
      glFramebufferTexture(GL_FRAMEBUFFER, GL_COLOR_ATTACHMENT0, tex, 0);D;
      GLenum buffers[1] = { GL_COLOR_ATTACHMENT0 };
      glDrawBuffers(1, buffers);D;
    }
55
    void plot_finish(struct plot *s) {
      (void) s;
      glBindFramebuffer(GL_FRAMEBUFFER, 0);D;
    }
60
    void plot_do(struct plot *s, int count) {
      glUseProgram(s->program);D;
      glBindVertexArray(s->vao);D;
      glDrawArrays(GL_POINTS, 0, count);D;
65    glBindVertexArray(0);D;
      glUseProgram(0);D;
    }
```

46 src/plot_frag.glsl

```
flat in vec3 g;
out layout(location = 0, index = 0) vec4 c;
void main() {
  c = vec4(g, 1.0);
5 }
```

47 src/plot_vert.glsl

```
uniform mat4 mvp;
in vec4 g0;
in vec2 q0;
flat out vec3 g;
5 void main() {
  g = g0.xyz;
  gl_Position = mvp * vec4(q0, 0.0, 1.0);
}
```

48 src/preamble.glsl

```
#version 330 core
precision highp float;
```

49 src/prune.c

```
struct prune {
  GLuint program;
  GLuint vao;
  GLuint query;
5 };
```

```
void prune_begin(struct prune *s, GLuint vbo, struct dvariable *d) {
  s->program = 0;
  s->vao = 0;
  s->query = 0;

  int len = 65536;
  char *src = (char *) calloc(1, len);
  int slen = 0;
#define L0(str) slen += snprintf(src + slen, len - slen - 1, str)
#define L1(fmt,arg) slen += snprintf(src + slen, len - slen - 1, fmt, arg)
#define L2(fmt,arg1,arg2) slen += snprintf(src + slen, len - slen - 1, fmt, arg1
    ↳ , arg2)

  for(int i = 0; i < d->packed; ++i) {
    L1("in vec4 p%d_i;\n", i);
  }
  L0("in vec4 g_i;\n");
  L0("in vec2 q_i;\n");
  for(int i = 0; i < d->packed; ++i) {
    L1("flat out vec4 p%d_v;\n", i);
  }
  L0("flat out vec4 g_v;\n");
  L0("flat out vec2 q_v;\n");
  L0("void main() {\n");
  for(int i = 0; i < d->packed; ++i) {
    L2("  p%d_v = p%d_i;\n", i, i);
  }
  L0("  g_v = g_i;\n");
  L0("  q_v = q_i;\n");
  L0("}\n");
  char *prune_vert = src;

  src = (char *) calloc(1, len);
  slen = 0;
  L0("layout(points) in;\n");
  L0("layout(points, max_vertices = 1) out;\n");
  for(int i = 0; i < d->packed; ++i) {
    L1("flat in vec4 p%d_v[1];\n", i);
  }
  L0("flat in vec4 g_v[1];\n");
  L0("flat in vec2 q_v[1];\n");
  for (int i = 0; i < d->packed; ++i) {
    L1("flat out vec4 p%d_o;\n", i);
  }
  L0("flat out vec4 g_o;\n");
  L0("flat out vec2 q_o;\n");
  L0("void main() {\n");
  L0("  float umean = (g_v[0].y + 10.0) / (g_v[0].x + 1.0);\n");
  L0("  float lmean = (g_v[0].y - 10.0) / (g_v[0].x + 1.0);\n");
  L0("  float delta = abs(tanh(8.0 * umean) - tanh(8.0 * lmean));\n");
  L0("  if (! (g_v[0].x >= 64.0 && delta < 0.5 / 256.0)) {\n");
  for (int i = 0; i < d->packed; ++i) {
    L2("  p%d_o = p%d_v[0];\n", i, i);
  }
  L0("    g_o = g_v[0];\n");
  L0("    q_o = q_v[0];\n");
```

```
    L0("    EmitVertex();\n");
    L0("    EndPrimitive();\n");
    L0("  }\n");
    L0("}\n");
    char *prune_geom = src;

#undef L0
#undef L1
#undef L2

    s->program = compile_program("prune", prune_vert, prune_geom, 0, d->nvaryings,
        d->varyings);
    glGenVertexArrays(1, &s->vao);D;
    glGenQueries(1, &s->query);D;
    glBindVertexArray(s->vao);D;
    glBindBuffer(GL_ARRAY_BUFFER, vbo);D;
    dvariable_setpointers(d, s->program);
    glBindBuffer(GL_ARRAY_BUFFER, 0);D;
    glBindVertexArray(0);D;
}

void prune_end(struct prune *s) {
    glDeleteProgram(s->program);D;
    s->program = 0;
    glDeleteVertexArrays(1, &s->vao);D;
    s->vao = 0;
    glDeleteQueries(1, &s->query);D;
    s->query = 0;
}

void prune_do(struct prune *s, GLuint vbo, GLuint *count) {
    glEnable(GL_RASTERIZER_DISCARD);
    glBindVertexArray(s->vao);D;
    glUseProgram(s->program);D;
    glBindBufferBase(GL_TRANSFORM_FEEDBACK_BUFFER, 0, vbo);D;
    glBeginQuery(GL_TRANSFORM_FEEDBACK_PRIMITIVES_WRITTEN, s->query);D;
    glBeginTransformFeedback(GL_POINTS);D;
    glDrawArrays(GL_POINTS, 0, *count);D;
    glEndTransformFeedback();D;
    glEndQuery(GL_TRANSFORM_FEEDBACK_PRIMITIVES_WRITTEN);D;
    glGetQueryObjectuiv(s->query, GL_QUERY_RESULT, count);D;
    glBindBufferBase(GL_TRANSFORM_FEEDBACK_BUFFER, 0, 0);D;
    glUseProgram(0);D;
    glBindVertexArray(0);D;
    glDisable(GL_RASTERIZER_DISCARD);
}
```

50 src/record.c

```
#include "view.h"

struct record {
    unsigned char *buffer;
    int bytes;
    int seqno;
    bool screenshot;
    bool video;
```

```
  char *filename;
};

void record_begin(struct record *s) {
  s->buffer = 0;
  s->bytes = 0;
  s->seqno = 0;
  s->screenshot = false;
  s->video = false;
  s->filename = 0;
}

void record_end(struct record *s) {
  if (s->buffer) {
    free(s->buffer);
    s->buffer = 0;
    s->bytes = 0;
  }
}

void record_write(FILE *out, unsigned char *buffer, struct view *v) {
  fprintf(out, "P5\n");
  fprintf(out, "# lyapunov-fm/0\n");
  fprintf(out, "# geometry = %dx%d\n", v->width, v->height);
  fprintf(out, "# tiling   = %dx%d+%d+%d\n", v->tilesx, v->tilesy, v->tilex, v->tiley);
  fprintf(out, "# delay    = %f\n", v->delay);
  fprintf(out, "# radius   = %f\n", v->radius);
  fprintf(out, "# note_x   = %f\n", v->center[0]);
  fprintf(out, "# note_y   = %f\n", v->center[1]);
  fprintf(out, "# index_x  = %f\n", v->center[2]);
  fprintf(out, "# index_y  = %f\n", v->center[3]);
  fprintf(out, "# matrix   = [ %f, %f, %f, %f ; %f, %f, %f, %f ]\n"
    , v->matrix[0][0], v->matrix[0][1], v->matrix[0][2], v->matrix[0][3]
    , v->matrix[1][0], v->matrix[1][1], v->matrix[1][2], v->matrix[1][3]
    );
  fprintf(out, "%d %d\n255\n", v->width, v->height);
  fflush(out);
  for (int y = v->height - 1; y >= 0; --y) {
    fwrite(buffer + y * v->width, v->width, 1, out);
  }
  fflush(out);
}

void record_do(struct record *s, struct view *v) {
  if (s->screenshot || s->video) {
    if (! s->buffer) {
      s->bytes = v->width * v->height;
      s->buffer = (unsigned char *) malloc(s->bytes);
    }
    glReadPixels(0, 0, v->width, v->height, GL_RED, GL_UNSIGNED_BYTE, s->buffer);
    if (s->screenshot) {
      char filename[1024];
      snprintf(filename, 1000, "%04d.pgm", s->seqno++);
      FILE *out = fopen(s->filename ? s->filename : filename, "wb");
      if (! out) {
```

```
        fprintf(stderr, "\aERROR: screenshot failed\n");
      } else {
        record_write(out, s->buffer, v);
        fclose(out);
      }
      s->screenshot = false;
    }
    if (s->video) {
      record_write(stdout, s->buffer, v);
    }
  }
}
```

51 src/render.c

```
void render_begin(struct render *s, int d, float df) {
  s->reshape_requested = 0;
  s->win_width = -1;
  s->win_height = -1;
  s->tex_size = -1;
  s->vbo_size = -1;
  s->tex = 0;
  s->fbo = 0;
  s->vbo[0] = 0;
  s->vbo[1] = 0;
  s->count = 0;
  s->steps = 0;
  s->samplerate = 48000.0;

  glClampColor(GL_CLAMP_VERTEX_COLOR, GL_FALSE);D;
  glClampColor(GL_CLAMP_READ_COLOR, GL_FALSE);D;
  glClampColor(GL_CLAMP_FRAGMENT_COLOR, GL_FALSE);D;
  glGenTextures(1, &s->tex);D;
  glBindTexture(GL_TEXTURE_2D, s->tex);D;
  glTexParameteri(GL_TEXTURE_2D, GL_TEXTURE_MIN_FILTER, GL_NEAREST);D;
  glTexParameteri(GL_TEXTURE_2D, GL_TEXTURE_MAG_FILTER, GL_NEAREST);D;
  glBindTexture(GL_TEXTURE_2D, 0);D;
  glGenFramebuffers(1, &s->fbo);D;
  glGenBuffers(2, s->vbo);D;

  dvariable_begin(&s->dvariable, d);
  fillc_begin(&s->fillc);
  init_begin(&s->init, &s->dvariable);
  step_begin(&s->step, s->vbo[0], &s->dvariable, df);
  prune_begin(&s->prune, s->vbo[1], &s->dvariable);
  plot_begin(&s->plot, s->vbo[1], &s->dvariable);
  colour_begin(&s->colour);

  s->idle.f = 0;
  s->done.f = 0;
  s->done.u = 0;
}

void render_end(struct render *s) {
  glDeleteTextures(1, &s->tex);D;
  glDeleteFramebuffers(1, &s->fbo);D;
  glDeleteBuffers(2, s->vbo);D;
```

```
  fillc_end(&s->fillc);
  init_end(&s->init);
  step_end(&s->step);
  prune_end(&s->prune);
  plot_end(&s->plot);
  colour_end(&s->colour);
}

void render_reshape(struct render *s, int w, int h) {
  s->win_width  = w;
  s->win_height = h;
  if (s->reshape_requested) {
    s->reshape_requested = 0;
    s->view.width  = w;
    s->view.height = h;
  }
}

void render_go(struct render *s);
void render_do(struct render *s, struct view *v) {
  if (v->width != s->win_width || v->height != s->win_height) {
    if (! s->reshape_requested)  {
      s->reshape_requested = 1;
      glutReshapeWindow(v->width, v->height);
    }
    return;
  }
  int new_count = v->width * v->height;
  int new_size = ceil2n(max(v->width, v->height));
  bool reallocate = new_size != s->tex_size;
  if (reallocate) {
    s->tex_size = new_size;
    glBindTexture(GL_TEXTURE_2D, s->tex);D;
    glTexImage2D(GL_TEXTURE_2D, 0, GL_RGBA32F, s->tex_size, s->tex_size, 0, ↙
        ↳ GL_RGBA, GL_UNSIGNED_BYTE, 0);D;
    glBindTexture(GL_TEXTURE_2D, 0);D;
  }
  if (s->vbo_size != new_count) {
    s->vbo_size = new_count;
    for (int i = 0; i < 2; ++i) {
      glBindBuffer(GL_ARRAY_BUFFER, s->vbo[i]);D;
      glBufferData(GL_ARRAY_BUFFER, s->vbo_size * s->dvariable.stride, 0, ↙
          ↳ GL_DYNAMIC_COPY);D;
      glBindBuffer(GL_ARRAY_BUFFER, 0);D;
    }
  }
  view_copy(&s->view, v);
  s->count = s->view.width * s->view.height;
  s->steps = 0;
  s->total = 0;
  clock_gettime(CLOCK_MONOTONIC, &s->t_start);
  clock_gettime(CLOCK_MONOTONIC, &s->t_update);
  fillc_do(&s->fillc, s->vbo[1], s->fbo, s->tex, &s->view);
  init_do(&s->init, s->vbo[1], s->vbo[0], &s->view);
  step_start(&s->step, s->samplerate, &s->view);
  s->idle.f = render_go;
  glutIdleFunc(idlecb);
```

```
}

void render_go(struct render *s) {
  plot_start(&s->plot, s->fbo, s->tex, &s->view);

#define GO \
  s->total += s->count; \
  step_do(&s->step, s->vbo[1], s->count, s->steps++ > 128); \
  plot_do(&s->plot, s->count); \
  prune_do(&s->prune, s->vbo[0], &s->count);

  glFinish();
  struct timespec t0;
  clock_gettime(CLOCK_MONOTONIC, &t0);

  GO

  glFinish();
  struct timespec t1;
  clock_gettime(CLOCK_MONOTONIC, &t1);

  double dt = time_difference(&t1, &t0);

  for (double t = dt; t < 0.04 && s->count /* && s->total < 8192 * s->view.width
      ↪  * s->view.height */ ; t += dt) {
    GO
  }

#undef GO

  plot_finish(&s->plot);
  if (! (s->count /* && s->total < 8192 * s->view.width * s->view.height */)) {
    s->idle.f = 0;
  }
}

void render_idle(struct render *s) {
  if (s->idle.f) {
    s->idle.f(s);
  } else {
    glutIdleFunc(0);
  }
  glutPostRedisplay();
  glutReportErrors();
}

void render_display(struct render *s) {
  struct timespec t;
  clock_gettime(CLOCK_MONOTONIC, &t);
  if (time_difference(&t, &s->t_update) > 1 || ! s->count) {
    fprintf(stderr, "\r%8d%12d    %4.3f", s->count, s->total, time_difference(&t
        ↪ , &s->t_start)); fflush(stderr);
    clock_gettime(CLOCK_MONOTONIC, &s->t_update);
    if (! s->count) {
      fprintf(stderr, "\n");
    }
  }
```

```
  colour_do(&s->colour, s->tex, s->tex_size, &s->view);
  glutSwapBuffers();
  if (! s->idle.f) {
    if (s->done.f) {
      s->done.f(s, s->done.u);
    }
  }
}

void render_ondone(struct render *s, void (*f)(struct render *, void *), void *u↙
    ↳ ) {
  s->done.f = f;
  s->done.u = u;
}
```

52 src/render.h

```
#ifndef RENDER_H
#define RENDER_H 1

#include "view.h"

struct idlefunc_t { void (*f)(struct render *); };
struct donefunc_t { void (*f)(struct render *, void *); void *u; };

struct render {
  struct dvariable dvariable;
  struct prune prune;
  struct idlefunc_t idle;
  struct donefunc_t done;
  double samplerate;
  int reshape_requested;
  int win_width;
  int win_height;
  int tex_size;
  int vbo_size;
  GLuint tex;
  GLuint fbo;
  GLuint vbo[2];
  GLuint count;
  int steps;
  int total;
  struct view view;
  struct fillc fillc;
  struct init init;
  struct step step;
  struct plot plot;
  struct colour colour;
  struct timespec t_start;
  struct timespec t_update;
};

void render_begin(struct render *s, int d, float df);
void render_end(struct render *s);
void render_reshape(struct render *s, int w, int h);
void render_do(struct render *s, struct view *v);
void render_idle(struct render *s);
```

```
void render_display(struct render *s);
void render_ondone(struct render s, void (*f)(struct render *, void *), void *u)↙
    ↳ ;

#endif
```

53 src/s2c.sh

```
#!/bin/bash
echo "static const char $1[] ="
sed 's|\\|\\\\|g' |
sed 's|"|\\"|g' |
sed 's|^|"|' |
sed 's|$|\\n"|'
echo ";"
```

54 src/shader.c

```
#include "preamble.glsl.c"

void debug_program(GLuint program, const char *name, const GLchar *vert, const ↙
    ↳ GLchar *geom, const GLchar *frag) {
  if (program) {
    GLint linked = GL_FALSE;
    glGetProgramiv(program, GL_LINK_STATUS, &linked);D;
    if (linked != GL_TRUE) {
      fprintf(stderr, "%s: link failed\n", name);
    }
    int length;
    glGetProgramiv(program, GL_INFO_LOG_LENGTH, &length);D;
    char *buffer = (char *) malloc(length + 1);
    glGetProgramInfoLog(program, length, NULL, buffer);D;
    buffer[length] = 0;
    if (buffer[0]) {
      fprintf(stderr, "==== %s.log ====\n%s\n", name, buffer);
      if (vert) { fprintf(stderr, "---- %s.vert ----\n%s\n", name, vert); }
      if (geom) { fprintf(stderr, "---- %s.geom ----\n%s\n", name, geom); }
      if (frag) { fprintf(stderr, "---- %s.frag ----\n%s\n", name, frag); }
    }
    free(buffer);
  } else {
    fprintf(stderr, "%s: program failed\n", name);
  }
}

void compile_shader(GLint program, GLenum type, const GLchar *source) {
  const GLchar *sources[2] = { preamble, source };
  GLuint shader = glCreateShader(type);D;
  glShaderSource(shader, 2, sources, 0);D;
  glCompileShader(shader);D;
  glAttachShader(program, shader);D;
  glDeleteShader(shader);D;
}

GLint compile_program(const char *name, const GLchar *vert, const GLchar *geom, ↙
    ↳ const GLchar *frag, int nvaryings, GLchar **varyings) {
```

```
  GLint program = glCreateProgram();D;
  if (vert) { compile_shader(program, GL_VERTEX_SHADER   , vert); }
  if (geom) { compile_shader(program, GL_GEOMETRY_SHADER, geom); }
  if (frag) { compile_shader(program, GL_FRAGMENT_SHADER, frag); }
  if (nvaryings) {
    glTransformFeedbackVaryings(program, nvaryings, (const GLchar **) varyings, ↙
        ↳ GL_INTERLEAVED_ATTRIBS);D;
  }
  glLinkProgram(program);D;
  debug_program(program, name, vert, geom, frag);
  return program;
}

GLint compile_program(const char *name, const GLchar *vert, const GLchar *geom, ↙
    ↳ const GLchar *frag) {
  return compile_program(name, vert, geom, frag, 0, 0);
}
```

55 src/state.c

```
#include "render.h"
#include "view.h"
#include "audio.h"
#include "batch.h"

static struct {
  int mode;
  bool rendering;
  struct render render;
  struct record record;
  struct audio audio;
  struct batch batch;
} S;

void state_init(int mode, double delay) {
  S.mode = mode;
  batch_begin(&S.batch);
  view_init(&S.render.view, 1280, 720, 120, 120, 0, 0, 72, delay);
//  view_init(&S.render.view, 1800, 1200, 120, 120, 0, 0, 72, 0);
//  1400, 1000, 123.609177, 72.565994, 17.597776, -21.296762, 3, 1);

// 183.863355798875289793, 183.863355798875289793, 1.306958840076017037, ↙
    ↳ 1.306958840076017037, 0.05);
//  125.525291442871093750, 125.525291442871093750, -21.048765182495117188, ↙
    ↳ 2.741522789001464844, 1.125000000000000000);
//148.668530260743807503, 148.668530260743807503, 1.812859159382673724, ↙
    ↳ 1.812859159382673724, 0.198873782208716543/4);
//118.093467857212488070, 113.307444175989090240, 11.232046341461641603, ↙
    ↳ -32.543675944390649590, 4.242640687119273224);
  S.rendering = false;
}
```

56 src/step.c

```
struct step {
  GLuint program;
```

```
  GLint sr;
  GLint iters;
  GLint perturb;
  GLint dither;
  GLint model;
  GLint center;
  GLint radius;
  GLint keep;
  GLuint vao;
  float dithersize;
  int d;
  GLfloat *noise;
};

void step_begin(struct step *s, GLuint vbo, struct dvariable *d, float df) {
  s->program = 0;
  s->sr = -1;
  s->iters = -1;
  s->perturb = -1;
  s->dither = -1;
  s->model = -1;
  s->center = -1;
  s->radius = -1;
  s->keep = -1;
  s->vao = 0;
  s->dithersize = 0;
  s->d = d->d;

  int len = 65536;
  char *src = (char *) calloc(1, len);
  int slen = 0;
#define L0(str) slen += snprintf(src + slen, len - slen - 1, str)
#define L1(fmt,arg) slen += snprintf(src + slen, len - slen - 1, fmt, arg)
#define L2(fmt,arg1,arg2) slen += snprintf(src + slen, len - slen - 1, fmt, arg1↙
    ↳ , arg2)

L1("const int D = %d;\n", d->d + 1);
L1("const float df = %f;\n", df);
L0("uniform float sr;\n");
L0("uniform int iters;\n");
L1("uniform vec2 perturb[%d];\n", d->d + 1);
L0("uniform vec2 dither;\n");
L0("uniform bool keep;\n");
L0("uniform mat4 model;\n");
L0("uniform vec4 center;\n");
L0("uniform float radius;\n");
for (int i = 0; i < d->packed; ++i) {
  L1("in vec4 p%d_i;\n", i);
}
L0("in vec4 g_i;\n");
L0("in vec2 q_i;\n");
for (int i = 0; i < d->packed; ++i) {
  L1("flat out vec4 p%d_o;\n", i);
}
L0("flat out vec4 g_o;\n");
L0("flat out vec2 q_o;\n");
L0("float delta(vec2 l, vec2 r) {\n");
```

```
  L0("  vec2 d = l - r;\n");
  L0("  d = abs(d);\n");
  L0("  d = min(d, abs(d - vec2(1.0)));\n");
  L0("  return dot(d, d);\n");
  L0("}\n");
  L0("vec2 mtoi(vec2 m) {\n");
  L0("  return 440.0 * pow(vec2(2.0), (m - vec2(69.0)) / 12.0) / sr;\n");
  L0("}\n");
  L0("float wmix(float x, float y, float f) {\n");
  L0("  if (y < x) { return mix(x, y + 1.0, f); }\n");
  L0("  else       { return mix(x, y, f); }\n");
  L0("}\n");
  L0("vec2 wmix(vec2 x, vec2 y, float f) {\n");
  L0("  return vec2(wmix(x.x, y.x, f), wmix(x.y, y.y, f));\n");
  L0("}\n");
  L0("\n");
  L0("vec2 step(vec2 z, vec2 zz, vec4 a1) {\n");
  L0("  return fract(z.xy + mtoi(a1.xy + a1.zw * cos(6.283185307179586 * zz.yx)))
      ↪ ;\n");
  L0("}\n");
  L0("void main() {\n");
  L0("// state vector\n");
  L0("  vec2 l[D];\n");
  L0("  vec2 r[D];\n");
  L0("\n");
  L0("  // unpack state vector\n");
  for (int i = 0; i < d->packed; ++i) {
    L2("  r[%d] = p%d_i.xy;\n", 2 * i, i);
    if (2 * i < d->d) {
      L2("  r[%d] = p%d_i.zw;\n", 2 * i + 1, i);
    }
  }
  L0("\n");
  L0("  // perturb\n");
  L0("  for (int i = 0; i < D; ++i) {\n");
  L0("    l[i] = r[i] + perturb[i];\n");
  L0("  }\n");
  L0("\n");
  L0("  // maximum separation\n");
  L0("  float sM = float(D) * 0.25;\n");
  L0("  // initial separation\n");
  L0("  float s0 = 0.0;\n");
  L0("  for (int i = 0; i < D; ++i) {\n");
  L0("    s0 += delta(l[i], r[i]);\n");
  L0("  }\n");
  L0("\n");
  L0("  // parameter\n");
  L0("  vec4 a1 = model * (vec4(radius * (q_i + dither), 0.0, 0.0)) + center;\n");
  L0("\n");
  L0("  // step\n");
  L0("  vec3 f = vec3(g_i.xyz);\n");
  L0("  int j = 0;\n");
  L0("  for (int h = 0; h < iters/D + 1; ++h) {\n");
  L0("    if (s0 > sM * 0.25) { break; }\n");
  L0("    for (int i = 0; i < D; ++i) {\n");
  L0("      int jj = (j + 1) %% D;\n");
  L0("      int jjj = (j + 2) %% D;\n");
```

```
115   L0("      l[jj] = step(l[j], wmix(l[jj], l[jjj], df), a1);\n");
      L0("      r[jj] = step(r[j], wmix(r[jj], r[jjj], df), a1);\n");
      L0("      j = jj;\n");
      L0("    }\n");
      L0("\n");
120   L0("    // final separation\n");
      L0("    float s1 = 0.0;\n");
      L0("    for (int i = 0; i < D; ++i) {\n");
      L0("      s1 += delta(l[i], r[i]);\n");
      L0("    }\n");
125   L0("    // accumulate lyapunov exponent\n");
      L0("    if (keep && s0 > 0.0 && s1 > 0.0) {\n");
      L0("      float k = 0.5 * log(s1 / s0) / float(D);\n");
      L0("      if (! ( 10.0 >= k)) { k =  10.0; }\n");
      L0("      if (! (-10.0 <= k)) { k = -10.0; }\n");
130   L0("      f += vec3(1.0, k, k * k);\n");
      L0("    }\n");
      L0("    s0 = s1;\n");
      L0("  }\n");
      L0("  g_o = vec4(f, g_i.w);\n");
135   L0("\n");
      L0("  // pass through pixel coords\n");
      L0("  q_o = q_i;\n");
      L0("\n");
      L0("  // pack state vector\n");
140   L0("  // explore more orbits by picking the perturbed version\n");
      for (int i = 0; i < d->packed; ++i) {
        L2("  p%d_o.xy = l[(j + %d) %% D];\n", i, 2 * i);
        if (2 * i < d->d) {
          L2("  p%d_o.zw = l[(j + %d) %% D];\n", i, 2 * i + 1);
145     }
      }
      L0("\n");
      L0("}\n");

150   #undef L0
      #undef L1
      #undef L2

        GLchar *step_vert = src;
155
        s->program = compile_program("step", step_vert, 0, 0, d->nvaryings, d->↙
            ↳ varyings);
        s->sr = glGetUniformLocation(s->program, "sr");D;
        s->iters = glGetUniformLocation(s->program, "iters");D;
        s->perturb = glGetUniformLocation(s->program, "perturb");D;
160     s->noise = (GLfloat *) calloc(1, 2 * (s->d + 1) * sizeof(GLfloat));
        s->dither = glGetUniformLocation(s->program, "dither");D;
        s->model = glGetUniformLocation(s->program, "model");D;
        s->center = glGetUniformLocation(s->program, "center");D;
        s->radius = glGetUniformLocation(s->program, "radius");D;
165     s->keep = glGetUniformLocation(s->program, "keep");D;
        glGenVertexArrays(1, &s->vao);D;
        glBindVertexArray(s->vao);D;
        glBindBuffer(GL_ARRAY_BUFFER, vbo);D;
        dvariable_setpointers(d, s->program);D;
170     glBindBuffer(GL_ARRAY_BUFFER, 0);D;
```

```
  glBindVertexArray(0);D;
}

void step_end(struct step *s) {
  glDeleteProgram(s->program);D;
  s->program = 0;
  glDeleteVertexArrays(1, &s->vao);D;
  s->vao = 0;
}

void step_start(struct step *s, double samplerate, struct view *v) {
  glUseProgram(s->program);D;
  glUniform1f(s->sr, samplerate);D;
  glUniform1i(s->iters, 360);D;
  glUniformMatrix4fv(s->model, 1, GL_FALSE, &v->matrix[0][0]);D;
  glUniform4fv(s->center, 1, &v->center[0]);D;
  glUniform1f(s->radius, v->radius);D;
  s->dithersize = 2.0f / (v->height * v->tilesy);
  glUseProgram(0);D;
}

void step_do(struct step *s, GLuint vbo, GLuint count, int keep) {
  glEnable(GL_RASTERIZER_DISCARD);
  glBindVertexArray(s->vao);D;
  glUseProgram(s->program);
  glUniform1i(s->keep, keep);D;
#define NOISE (0.00001 * (rand() / (double) RAND_MAX - 0.5))
  for (int i = 0; i < 2 * (s->d + 1); ++i) {
    s->noise[i] = NOISE;
  }
  glUniform2fv(s->perturb, s->d + 1, s->noise);D;
#undef NOISE
#define NOISE ((rand() / (double) RAND_MAX - 0.5) * s->dithersize)
  glUniform2f(s->dither, NOISE, NOISE);D;
#undef NOISE
  glBindBufferBase(GL_TRANSFORM_FEEDBACK_BUFFER, 0, vbo);D;
  glBeginTransformFeedback(GL_POINTS);D;
  glDrawArrays(GL_POINTS, 0, count);D;
  glEndTransformFeedback();D;
  glBindBufferBase(GL_TRANSFORM_FEEDBACK_BUFFER, 0, 0);D;
  glUseProgram(0);D;
  glBindVertexArray(0);D;
  glDisable(GL_RASTERIZER_DISCARD);
}
```

57 src/utility.c

```
template <typename T> T& max(T& a, T& b) {
  return a > b ? a : b;
}

int ceil2n(int z) {
  int n = 1;
  while (0 < n && n < z) {
    n <<= 1;
  }
  return n;
```

```
}

double time_difference(const struct timespec *t1, const struct timespec *t0) {
  return (t1->tv_sec - t0->tv_sec) + (t1->tv_nsec - t0->tv_nsec) / 1000000000.0;
}
```

58 src/view.c

```
void view_init(struct view *v, int width, int height, double centerx, double centery, double centerz, double centerw, double radius, double delay) {
  v->width = width;
  v->height = height;
  v->matrix = glm::mat4(0.0f);
  v->matrix[0][0] = 1;
  v->matrix[0][1] = 1;
  v->matrix[1][2] = 1;
  v->matrix[1][3] = 1;
  v->center[0] = centerx;
  v->center[1] = centery;
  v->center[2] = centerz;
  v->center[3] = centerw;
  v->radius = radius;
  v->tilex = 0;
  v->tiley = 0;
  v->tilesx = 1;
  v->tilesy = 1;
  v->delay = delay;
}

void view_copy(struct view *dst, struct view *src) {
  dst->width = src->width;
  dst->height = src->height;
  dst->matrix = src->matrix;
  dst->center = src->center;
  dst->radius = src->radius;
  dst->tilex = src->tilex;
  dst->tiley = src->tiley;
  dst->tilesx = src->tilesx;
  dst->tilesy = src->tilesy;
  dst->delay = src->delay;
}

void view_reshape(struct view *v, int width, int height) {
  v->width = width;
  v->height = height;
}

void view_reset(struct view *v, double centerx, double centery, double centerz, double centerw, double radius) {
  v->center[0] = centerx;
  v->center[1] = centery;
  v->center[2] = centerz;
  v->center[3] = centerw;
  v->radius = radius;
}

void view_plane(struct view *v, int which) {
```

```
  v->matrix = glm::mat4(0.0f);
  v->matrix[0][0] = 1;
  v->matrix[0][1] = which;
  v->matrix[1][2] = 1;
  v->matrix[1][3] = which;
}

void view_coord(glm::vec4 *c, struct view *v, int px, int py) {
  glm::vec4 p = glm::vec4(float((px - v->width / 2.0) * 2.0 / v->height), float
      ↪ ((v->height / 2.0 - py) * 2.0 / v->height), 0.0f, 0.0f);
  *c = v->matrix * (v->radius * p) + v->center;
}

void view_center(struct view *v, glm::vec4 &c) {
  v->center = c;
}

void view_zoom(struct view *v, glm::vec4 &c, float z) {
  float g = pow(0.5, z);
  v->center = g * v->center + (1 - g) * c;
  v->radius *= g;
}

void view_zoom(struct view *v, float z) {
  view_zoom(v, v->center, z);
}
```

59 src/view.h

```
#ifndef VIEW_H
#define VIEW_H 1

struct view {
  int width;
  int height;
  glm::mat4 matrix;
  glm::vec4 center;
  float radius;
  int tilex, tiley, tilesx, tilesy;
  float delay;
};

void view_init(struct view *v, int width, int height, double centerx, double
    ↪ centery, double centerz, double centerw, double radius, double delay);
void view_copy(struct view *dst, struct view *src);
void view_reshape(struct view *v, int width, int height);
void view_reset(struct view *v, double centerx, double centery, double centerz,
    ↪ double centerw, double radius);
void view_plane(struct view *v, int which);
void view_coord(glm::vec4 *c, struct view *v, int px, int py);
void view_center(struct view *v, glm::vec4 &c);
void view_zoom(struct view *v, float z);
void view_zoom(struct view *v, glm::vec4 &c, float z);

#endif
```